THEORIA

A SWEDISH JOURNAL
OF PHILOSOPHY AND PSYCHOLOGY

FOUNDED BY ÅKE PETZÄLL

EDITED BY
KONRAD MARC-WOGAU

●

MANAGING EDITOR: K. MARC-WOGAU

CONSULTING EDITORS:
JUSTUS HARTNACK, K. E. TRANØY, G. H. v. WRIGHT

●

SUB-EDITOR: MANFRED MORITZ

●

VOLUME XXV 1959

LUND

C. W. K. GLEERUP
LUND

EJNAR MUNKSGAARD
COPENHAGEN

BERLINGSKA BOKTRYCKERIET
LUND 1959

CONTENTS:

Articles:

Discussions:

Reviews:

Bibliographical Notes:

The Interpretation of Formalized Implication

by

HASKELL B. CURRY
(Pennsylvania State University)

This paper discusses the nature and meaning of implication as it appears in certain formalized deductive theories, and contains, along with this, considerable discussion of the nature of these deductive theories themselves. These theories are mathematical in nature;[1] but they nevertheless give rise to ideas of some philosophical interest. These ideas do not appear to be as well understood among philosophers as they ought to be; perhaps this is because some of the conclusions are buried under a mass of mathematical detail. In this paper I shall attempt an exposition which is as free as possible from mathematical technicalities. The object is to make a breach in some of the barriers between mathematics and philosophy.

It is important that certain limitations of this article be understood. This is not an essay on the philosophy of implication per se. It is not concerned with what words expressing implication in ordinary language actually mean, nor with "explicating" the most general circumstances under which one statement may be inferred from another.[2] By "formalized implication" I mean an implication relation which is explicitly formulated as a constituent of a formalized propositional algebra. We are concerned with meanings which can be associated with such a formalized implication, and the relationship of those meanings to the formal

[1] For example, those developed in [TFD] and [LLA]. (For explanation of the letters in brackets see the Bibliography.) The ideas are due to a number of persons. References to the sources, supplementing those given here, will be found in the papers cited.

[2] For this see Tarski [BLF].

structure. Again, the nonmathematical character of this exposition imposes some further limitations. If it were possible to completely understand a mathematical result without understanding the mathematics, then the latter would be unnecessary. A reader who wishes thorough understanding will no doubt find that this treatment needs to be supplemented by consulting more technical presentations.[3] Finally I am unable to take account of certain philosophical technicalities, since these are outside my field of competence.

The plan of the paper is as follows. We begin, in § 1, with an account of formalized deductive systems in general. This account has to be made rather full in order to form a basis for what follows; and there is evidence of some misunderstanding of my previous writings on this topic. The account is specialized in § 2 to systems of the same type as propositional algebra;[4] at the same time some general conventions are stated. In § 3 we consider the nature of implication, as an operation in the uninterpreted algebra. The interpretations of the algebra are discussed in § 4; and the interpretation of implication in § 5. Finally, in § 6, there are a few remarks concerning the meaning of "strict" implication.

A few general comments must be made before we proceed. In the first place the terms 'proposition', 'statement', 'sentence', and 'clause' are used in this paper in technical senses. These senses will not be explained fully until § 4; the usage in the meantime is probably self-explanatory, but the reader is warned that the precise definitions, when made, may differ from those he has seen elsewhere. The usage is different, in particular, from some of my previous writings; this is intended as a correction. Again we are deeply concerned here with the nature of propositional algebra as such. Indeed one can say that implication is the basic notion of propositional algebra. The introduction of the other

[3] See those cited in footnote 1. Special references are made later.

[4] This term is preferred to 'calculus' because there are no bound variables. Cf. [LLA], p. 33. Although the significance of 'algebra' in connection with logic has changed a great deal since [LLA] was written, yet the term is retained for the sake of consistency.

positive notions would cause no particular difficulty, while negation is a separate subject.[5]

1. *Formal systems.*[6] There are two different sorts of system in the current literature to which the term 'formal[7] deductive system' can be applied. I shall first describe the features in which they are alike; then I shall discuss their differences and interrelations.

A deductive system is essentially an inductively generated set of statements called the (elementary)[8] *theorems* of the system. This is meant in the following sense. The specifications of the system first define a set of statements called the *elementary statements*. Next there is defined a subset of the elementary statements, called the *axiomatic statements*, which are asserted outright to be theorems. Finally there is given a set of *rules* for generating further theorems from those already obtained. Each of these rules has the following character: given a set of elementary statements (the premises) satisfying certain specified conditions, the rule associates to them an elementary statement (the conclusion); in the stricter forms of system the conclusion is uniquely determined once the premises are given. The elementary theorems of the system are then those generated from the axiomatic statements by means of the rules. That is, an elementary statement is a theorem exactly when it is the last of a sequence of such statements, each of which either is axiomatic or is the conclusion of an instance of one of the rules for which the premises of that same instance occur earlier in the sequence.

What makes such a system formal is its objective, definite cha-

---

[5] This is treated in [TFD] Chapter IV. The introductory parts of that chapter are nontechnical. See also [LLA] Chapter IV.

[6] On the subject of this section see [CLg] Chapter 1. A detailed account of the history is given there § 1S1, pp. 33–35. For recent additions see [CFS], [DFS].

[7] The term 'formal' is used in the preliminary discussion in a nontechnical sense. In the technical sense used later we shall see that a syntactical deductive system is not formal unless it is tectonic. Cf. note 19.

[8] Theorems which are not elementary are called epitheorems. For these see [CLg] Chapter 2. Since epitheorems do not occur here, the word 'elementary' will be omitted in connection with 'theorem'.

racter. The conventions must be such that it is always effectively ascertainable 1) whether a statement is elementary, 2) whether an elementary statement is axiomatic, and 3) whether an alleged application of one of the rules is correct. It may or may not be the case that, given an elementary statement, one can decide effectively whether or not it is a theorem; but given an alleged proof (with the necessary auxiliary information)[9] one can decide effectively whether or not it is correct. The objectiveness should be such that the decisions could be made, in principle, by a machine.[10] There is no objection to an infinite number of constituents if these requirements are fulfilled.[11]

The first requirement of definiteness is generally met in the following way. One specifies a set of admissible objects and certain admissible predicates;[12] then an elementary statement is one ascribing an admissible predicate to a certain sequence of the proper number of admissible objects. The nature of the admissible objects will concern us later; for the present we simply suppose they exist, and that it is effectively ascertainable whether an object is admissible or not. In the cases we are interested in, there is a single unary[13] predicate; this will be symbolized by the

---

[9] This auxiliary information would be what Kleene (in his [IMM], p. 87) calls an analysis. Church [ILM_2], n. 121, p. 53 requires that proofs be definite without any auxiliary information. Theoretically this is no restriction provided the number of possibilities is finite.

[10] In the language of recursive functions, if one accepts "Church's Thesis" (Kleene [IMM], § 62), the elementary and axiomatic statements each form a recursive set, while the theorems form a recursively enumerable set.

[11] Actually there is such an objection, but it is not relevant to the present problem. From the standpoint of ultimate foundations only finite systems are acceptable; but most work on foundations does not push the analysis that far. Strictly finite systems occur in combinatory logic, and in certain work of Post and Lorenzen. Cf. [CLg] §§ 1B2, 1E4, and Chapter 7; Rosenbloom [EML]; Lorenzen [EOL].

[12] The term 'predicate' is used here, following Hilbert and Ackermann, as including statement functions of any number of arguments. See their [GZT], 1st edition, p. 45, n. 1.

[13] For objections to the term 'singulary' used by certain logicians, see my review of Church [IML_2]. Note that 'first' and 'second' have been erroneously interchanged there. The term 'singular' would be acceptable.

Frege-sign '$\vdash$', used as a prefix; so that the elementary statements are those of the form

$$\vdash p, \tag{1}$$

i.e., they are expressed by sentences obtained from the expression (1) by putting the name of an object in the place of 'p'. In such a case the system is called *logistic*. Cases where there is a binary relation, expressed by such notations as '$=$', '$\leq$' used as infixes, are common in mathematics; more general possibilities, even those with an infinite number of predicates are conceivable: but these generalizations are not relevant here.

The study of a deductive system requires, like every other study, the use of language. Let us call the language used for this purpose the *U-language*.[14] This contains all the technical terms and symbols understood by both speaker and hearer. Of course this language is not absolutely fixed; one can adjoin new technical terms to it or otherwise modify it at any time. The definition of a new deductive system requires such additions: we need nouns to name the admissible objects and verbs to express the admissible predicates, as well as formation rules for stating how these elements are combined to form sentences expressing the elementary statements. We shall call this linguistic machinery collectively the *A-language*, and mean by 'U-language' the language understood prior to its introduction. Then the A-language is is adjoined to the U-language to be used therein; it is required that the A-language be so constituted that this can be done without confusion. The elementary statements can be expressed entirely in the A-language; but the rules require additional elements from the U-language. In the last analysis the definition

[14] The reader may prefer the term '*metalanguage*'. The term 'U-language' was introduced in [LFS]; for its relations to a 'metalanguage' see [LMF]. If the suggestion there made is adopted, then in a hierarchy of languages, each of the higher ones referring to those below it, the U-language will always be the top language, whereas the metalanguage will be the second from the bottom. Thus the U-language and the metalanguage coincide only when there are just two levels. In the case of an abstract formal system or an abstract syntax (defined later) the U-language will be the only language which enters.

of a deductive system is the introduction of an A-language with specifications for its use.

We turn now to consider the admissible objects. In this respect the two sorts of deductive system are different. In the first and more widely known sort, called here a *syntactical system*, or simply a *syntax*, the objects are the expressions of an object language. In the second sort, called here a *formal system* the objects are called obs; nothing is specified about them except that they are an inductively generated set. We shall discuss each of these two types of system somewhat more in detail.

We shall call the object language of a syntax its *O-language*. To define it one specifies simply a list (the O-alphabet) of atoms (the O-letters); it is then understood that any finite string (or series) of such O-letters, possibly with repetitions, is an *O-expression*. To form the elementary statements it is usually necessary to define a subclass of the O-expressions, here called *wefs* (i.e. well-formed O-expressions); then the elementary statements are formed by applying a predicate to a sequence of the proper number of wefs. The O-language is understood to be an uninterpreted language; and the rules for defining the system take into account only the "syntactical" properties of the O-expressions, i.e. the arrangement of the O-letters in them. In fact the O-letters do not have to be linguistic objects at all, in the sense of being in or capable of being adjoined to the U-language, provided that it is significant to reproduce them and to combine them into expressions. We can even have an *abstract syntax* in which no O-language is explicitly specified, but only the names for its expressions in the A-language. As for the A-language, one must devise some system of names which are linguistic objects and can reflect the possibility of combining the O-expressions by concatenation. If the O-letters are linguistic and are foreign to the U-language, one can of course take the O-expressions as their own names in the A-language; then the A-language is said to be *autonymous*. If the O-letters are linguistic but not foreign to the U-language, a common device, to avoid the confusion to which an autonymous A-language is then liable, is to use an O-expression in single quotes as A-name for that O-expression;

in this case one needs auxiliary technical devices in the U-language – such as Quine's "corners", – in order to express the rules and other statements which are not elementary.[14a]

The second type of deductive system is that for which priority [15] is claimed for the term 'formal system'. In this the objects are unspecified; to emphasize their arbitrariness they are called *obs*. It is specified that there are certain primitive obs (*atoms*), which can be identified and distinguished from one another in repeated occurrences; that there are certain operations for combining obs to form new obs; that the obs are generated from the atoms by the operations, so that to every ob there corresponds a construction which can be exhibited in the form of a tree (like a genealogical tree) with an operation indicated at each node; and that obs with different construction trees are distinct as obs, so that any identities between the construction trees must be derived explicitly in the theorems. The obs can thus be identified with the processes of construction so indicated. The rules take into account only those structural properties which can be determined from these trees without regard to the nature of the atoms and operations. All this amounts to introducing an A-language constructed as follows. First there are certain new symbols (the names of the atoms) which we agree to use as nouns; then there are certain *operators*, i.e. linguistic devices forming new A-nouns from previous ones, such that the mode of construction (in the sense of a tree) of such a complex A-noun is unique and definitely ascertainable. So long as these conditions are met it is irrelevant whether anything is said about what objects are admissible. A specification of these objects such that a unique object corresponds to each ob (and hence to each A-noun), and such that different objects are always assigned to

---

[14a] Such devices are hardly necessary in an informal discussion such as the present one. However single quotes are here reserved for this function of naming expressions, and double quotes are used for all other kinds of quotation marks, including Quine's corners.

[15] The term was introduced in [ALS]. For its development thereafter see [CLg] p. 34 f. Kleene [IMM] used the term for a syntax, and the term 'generalized arithmetic' for what is essentially a formal system.

distinct obs (or A-nouns), is called a *representation* of the system. Such a representation is to be distinguished from an interpretation (to be discussed later) in that it has no effect on the truth of the theorems.[16] A formal system without an assigned representation is called *abstract*; but, of course, the assignment of a representation, or a change of assignment once made, does not make the system any less definite and exact. Every formal system has a representation in the nouns of its A-language (autonymous r.); also a representation in numbers (Gödel r.). It is natural to conceive of a formal system as a structure invariant of changes in representation (and also of A-language); this does not require us to postulate a formal system as a Platonic abstractum, but simply to realize that we are regarding certain circumstances as nonessential.

It would be a mistake to suppose that the essential difference between these two notions is that the first is linguistic while the second is not. Neither sort of system is necessarily linguistic; either can be abstract. In either case the essential activity in the definition of the system is the adjunction of the A-language to the U-language to be *used* there in. What the nouns of this A-language denote is secondary. The principal difference lies in the requirement of unique construction for the obs of a formal system. This does not hold for a syntax. The O-expressions of a syntax are formed from the O-letters by concatenation; since that is an associative operation, there may be more than one process of construction. In all cases of any logical interest, however, this uniqueness of construction holds for the wefs with respect to certain operations which combine wefs to form new wefs. Such a syntax will be called *tectonic*.[17] A tectonic syntax, then, will satisfy all the requirements of a formal system if we take its wefs as obs. Conversely the autonymous representation of

---

[16] This is a consequence of the fact that the rules take into account only the structural properties. Cf. [CLg], § 2B. Note that an analogous statement can also be made for a syntax.

[17] For this term see [CFS], § 4. Nontectonic syntaxes occur in the work of Post, Markov, etc.

any formal system will necessarily be a tectonic syntax.[18] With nontectonic syntaxes we are not here concerned.[19]

Thus a formal system and a tectonic syntax are equivalent notions. In the following I shall use the terminology of an abstract formal system. A reader who prefers syntactical terms can easily supply a suitable tectonic O-language whose wefs are taken as the obs.

2. *Propositional algebra.* We now specialize these ideas to propositional algebra. This is a logistic formal system, i.e., it has a single unary predicate. We use the prefix '$\vdash$' for this, so that the elementary statements are of the form (1). In propositional algebra we shall use the term *'formula'* rather than the term 'ob'.[20] The atoms of propositional algebra are all indeterminates, i.e., nothing is said about them that is not true if arbitrary formulas are substituted for them. In some formulations this substitutivity is expressed by a rule; but it is well known that such a rule is superfluous if the axiomatic statements are stated in the form of "axiom schemes", such that any statement obtained from one of these schemes by substituting arbitrary formulas for the atoms is axiomatic. We shall suppose, therefore, that there is no rule of substitution in the algebra. Since we are not interested in the details of the formulation and have no explicit formal development, we have no use for symbols for the atoms or of any other particular formulas. We shall use the letters '*p*', '*q*', '*r*', etc., with or without subscripts, to designate unspecified formulas; these symbols are variables of the U-language for which arbitrary A-nouns can be substituted, and thus neither symbols

---

[18] So will any representation derived from it by certain types of substitution.

[19] I would say that such a system was not formal. It is easy, however, to construct a formal system of which it is a direct interpretation in the sense of § 4. Cf. [CFS] § 5.

[20] This convention is a compromise for the purposes of the present essay only. The word has too many conflicting uses to propose it as a permanent technical term. For strictly abstract purposes it would be better to retain the word 'ob'; but for practical purposes I prefer the term 'proposition'. Cf. footnote 39.

of some language being talked about nor names assigned to specific formulas.

We now make some special conventions which apply to any logistic formal system. We call the formulas p for which (1) is true the *assertible formulas,* and read (1) as "p is assertible". It is understood that if a complex A-noun is substituted for 'p' the prefix '$\vdash$' applies to the whole of the former without the need of explicit parentheses. If (1) is an axiomatic statement, p will be called an *axiomatic formula;* the term '*axiom*' will refer to either an axiomatic statement or an axiomatic formula according to the context. We shall also say that q follows from $p_1, \ldots, p_m$ by a rule R when $\vdash q$ follows from $\vdash p_1, \ldots, \vdash p_m$ by the rule R. Then, according to the definition of elementary theorem, (1) will be true if and only if there is a sequence $r_1, \ldots, r_n$ such that r_n is p, and every r_k is either an axiom or follows from some of the $r_1, \ldots, r_{k-1}$ by means of a rule.

Generalizing (1), let us define inductively a relation

$$p_1, \ldots, p_m \vdash q \tag{2}$$

between $m+1$ formulas ($m \geq 0$). We read (2) as "q is deducible from $p_1, \ldots, p_m$", or "$p_1, \ldots, p_m$ entail[21] q". It is defined to mean that q is generated from $p_1, \ldots, p_m$ and the axioms by the rules. This may be stated thus: There is a sequence $r_1, \ldots, r_n$ such that r_n is q and every r_k is either 1) an axiom, 2) one of the p_i, or 3) deducible some of the $r_1, \ldots, r_{k-1}$ by means of a rule. Then the assertible formulas are those which are deducible from nothing (so that (1) is a special case of (2)); and the formulas deducible from fixed premises $p_1, \ldots, p_m$ are precisely those which are assertible in the system obtained by adjoining $p_1, \ldots, p_m$ as additional axioms.

In the foregoing (1) and (2) are, of course, relative to the formulation of propositional algebra chosen. Let us call the basic system H, and let $H(p_1, \ldots, p_m)$ be the system obtained by adjoining $p_1, \ldots, p_m$ to H as additional axioms. Then what we have said

[21] Rosser [LMth] reads '$\vdash$' as 'yield'. This use of '$\vdash$' was proposed by Rosser in 1935. Note there is no confusion between the use of '$\vdash$' in (1) and (2) (see below).

is that (2) holds for H if and only if (1) holds for $H(p_1, \ldots, p_m)$.

3. *The implication operation.* We shall regard '$\supset$' as indicating an operation between formulas, i.e. such that, if p and q are formulas, $p \supset q$ is also a formula. From that standpoint the common reading "p implies q" for "$p \supset q$" is misleading and will are abandoned; we shall read "$p \supset q$" as "p ply q".[22]

The operation so indicated has the following property: Let H' be an extension of H (say by adjoining $r_1, \ldots, r_n$ as new axioms); then $p \supset q$ is assertible in H' if and only if q is deducible from p in H'. This property it is convenient to split into two properties (a) and (b), thus:

(a) If $\vdash p \supset q$ in H', then $p \vdash q$ in H'
(b) If $p \vdash q$ in H', then $\vdash p \supset q$ in H'

Here (a) is a way of stating the familiar rule of modus ponens. This is a rule of propositional algebra as ordinarily formulated; and we assume from now on that it is the sole rule of H. The property (b) is that asserted in the "deduction theorem". Henceforth we shall call $\supset$ an *implication* just when it satisfies (a) (or some suitable generalization of it); a *proper implication* just when (a) and (b) both hold.

For a proper implication let us consider the formula

(3) $$\vdash p \supset .\ (q \supset p).$$

This is regarded as paradoxical by many logicians. Thus Myhill in a recent paper [23] argues that intuitionistic propositional algebra cannot be interpreted in terms of strict implication because (3) would mean that a true proposition is strictly implied by any pro-

[22] The objection to "p implies q" is that '$\supset$' is not a verb and "$p \supset q$" is not a sentence. Likewise "if p then q" is objectionable because "p" and "q" are not sentences and '$\supset$' is not a conjunction. The reading "p then q" could be used if one can think of 'then' as an operator analogous to 'plus', but not if one insists on regarding it as a conjunction. Note that "p implies q" is a proper reading for "$\vdash p \supset q$" or "$p \vdash q$"; and the last again can be read "if $\vdash p$, then $\vdash q$".

[23] Myhill [ISg].

position, which is false if p is contingent. On the other hand (3) is certainly true for any proper implication. For (3) asserts that $q \supset p$ is assertible in the system H'(p) formed by adjoining p to H'; this in turn is equivalent to saying that p is assertible in the system H'(p, q) formed by adjoining q to H'(p). Since p is an axiom of H'(p, q), this is obviously true. In other words if p is deducible from nothing, then it is deducible from any premise. Far from being paradoxical, (3) is, from this standpoint, a platitude.

By methods somewhat similar to this Gentzen [24] showed that all the formulas for pure implication in the Heyting formulation of intuitionistic propositional algebra are assertible obs for any proper implication. Conversely the deduction theorem shows that the implication of the Heyting calculus – which, incidentally, is demonstrably consistent – is proper. Thus a formulation of intuitionistic pure implication has the properties (a), (b), and is the minimal system which does so.

Because of this minimal property it is appropriate to give to the implication expressed by the Heyting algebra the name '*absolute implication*'.[25] It follows from the foregoing that it is the implication which is defined, so to speak, by the properties (a) and (b). We shall use the designation 'HA' for a formulation of the positive Heyting algebra with modus ponens as the sole rule.

Implications which are weaker than the absolute have been considered in the literature. A notable example is the "weak implication" of A. Church.[26] This system can be motivated as follows.[27] The theory of absolute implication depends ultimately

[24] Gentzen [ULS]. See also note 30.

[25] It is appropriate also to apply the name 'absolute' to the positive Heyting algebra. In [TFD] it is given the symbol 'A'. This is the same as the "positive Logik" of Bernays; but that term seems misleading because there are positive theorems which are not absolute, e.g. Peirce's law (see (6), below).

[26] Church [WTI].

[27] This discussion is based in part on correspondence with Church in July, 1953. It contains a revision of comment (i) in the review by Craig and myself.

on the relation (2). Suppose we reinterpret (2) to mean that there is a derivation of q from $p_1, \ldots, p_m$ such that none of the premises $p_1, \ldots, p_m$ can be omitted from the proof without invalidating it. Then the implication arrived at by the methods of this section would be his implication, for which (3) fails. In that case, however, given fixed $p_1, \ldots, p_m$, the q for which (2) holds are not the set of consequences of $p_1, \ldots, p_m$ as that is ordinarily understood; indeed they form a set which is not ordinarily considered in deductive methodology at all.[28] Consequently it is appropriate to call such an implication "weakened", meaning that it is weaker than the absolute.[29]

Further light is shed on the nature of the absolute algebra by a second formulation due to Gentzen, namely his L-rules. By means of this it can be shown that the absolute algebra is decidable; further, that it forms the natural formulation for the relation of formal deducibility with respect to a broad class of formal systems. These matters will not be discussed further here.[30]

4. *The interpretation of propositional algebra.* Up to the present we have been concerned with propositional algebra as an uninterpreted calculus. We now turn to the question of interpretation. Our purpose will not be to argue for or against a particular method of interpretation, but rather to discuss certain general principles regarding interpretation as such. A part of the thesis will be that we are not committed to certain philosophical

[28] We have something similar in Klein's Erlangen Programm, where an affine theorem is one in which an affine postulate is essentially involved. But it is not quite the same concept, for an affine theorem may not involve all of the projective axioms. An explanation of affine geometry can be made in terms of absolute implication just as conveniently as in terms of Church's implication.

[29] Church's system appears to have been motivated by the desire to form a minimal logic in the sense of his abstract [MiL]. For discussion of this point and other examples of weakened logics see [GDT].

[30] For a detailed discussion see [TFD]. A summary is given in [LLA] III 6. For philosophers I attempted a very brief statement in [PBP]. The latter paper requires correction in that the statement, on page 150, that the L-system is decidable whenever the basic system is, does not represent what was intended; it has been shown simply that L is decidable when $\mathfrak{H}$ is void, which is the case here.

positions which have received some prominence in recent discussions.

Our first step will be to get a clear idea of what an interpretation is.[31] Given a formal system S, we speak of an interpretation of S whenever we have a correspondence between the elementary statements of S and certain statements known to us independently of S. Let us call the latter *contensive*[32] statements. The correspondence may be established in many different ways. It may happen that to each ob of S there is assigned a contensive[33] object, and to each predicate a contensive property or relation; in such a case we have a *direct interpretation*.[34] But an interpretation may be established in more elaborate ways than this. The important consideration is that the statements (or at any rate some of them)[35] are interpreted; what happens to the obs is irrelevant. The interpretation is *valid* just when all the contensive statements which correspond to the elementary theorems are true.[36]

For the propositional algebra H we specialize the notion of interpretation. We suppose given a class P of contensive objects, in which there are defined operations analogous to those in H, and a subclass T of P. We call the objects of P *propositions*. Then any assignment of propositions to the atoms of S transforms a formula p into a proposition p'. Given such an assignment, we can interpret (1) as the statement that p' is in T. We call this a *fixed interpretation*; it is of course a direct one. Another way of interpreting (1) is to associate to it the statement that p' is in T for every assignment. This, the *tautologous interpretation*, is

---

[31] See [CLg] § 1C3 and papers there cited.

[32] This term was introduced in [APM]; see also [CLg], l.c. Contensive statements are expressible entirely in the U-language (without the A-language).

[33] Defined analogously to 'contensive statement'.

[34] In a direct interpretation the same object can be assigned to different obs. In this it differs from a representation.

[35] More complex interpretations are conceivable, and are presumably important in physical theories. Cf. [TEx]; also Braithwaite [SEx].

[36] Note that validity, like many semantical concepts, is indefinite.

not direct. Other sorts of interpretation, involving, perhaps, restrictions of the assignments, are also conceivable.

Evidently we can form a great variety of interpretations for which the use of the term 'proposition' would seem rather strange. Thus we can find interpretations in which propositions are numbers, classes, points on a diagram, columns of a matrix, etc. These generalized interpretations do not concern us here. Our problem is interpretations which are related in some way to the traditional meaning of 'proposition', viz. the "content" or "meaning" of a statement. It is precisely the nature of this meaning which requires explanation.

In the above it was said that the propositions are objects. Nothing more is implied by this than that the expressions which stand for propositions function in the U-language as nouns. By contrast a statement is expressed in the U-language by a sentence. Here 'noun' and 'sentence' are names of fundamental linguistic categories which are taken as understood. Let us call the nouns used as names of propositions *clauses*. Then one can, if one likes, translate what is said here about propositions and statements into an equivalent discourse about clauses and sentences. We are using, to be sure, a somewhat idealistic terminology; but one can regard that as purely a matter of convenience.

After these preliminaries we return to the notion that a proposition is the meaning of a statement. We construe this as postulating a correspondence between statements and propositions; to every statement there corresponds a proposition which it asserts, and to every proposition there corresponds a statement which asserts it. Our task is to explain the nature of this correspondence.

It follows from the above that this manner of speaking does not commit us to a philosophy which regards these propositions as entities of an esoteric sort. It is sufficient to maintain that for every sentence there is a corresponding clause and vice versa. All the natural languages, at least the better developed ones, have devices for converting sentences into clauses. Thus the words

4) Anne is happy

form a sentence; on the other hand the expressions

(5) Anne's being happy
that Anne is happy
Anne's happiness

are cluases [37] which refer to precisely the same state of beatitude. We can say that (4) asserts the proposition named by the various clauses (5) without thereby implying that there is an object labelled "Happiness of Anne" roaming the universe. Regardless of our views as to the ontology of propositions, we can make sense out of the above interpretation provided we can understand the correspondence between sentences and clauses.

Many persons favor the abolition of the term 'proposition' altogether. They argue that what are here called propositions are really sentences. Then clauses are the names of sentences. But there is evidently a distinction between the sentences which are asserted and those which are mentioned,[38] and we can use the term 'proposition' to designate the latter. Frequently – indeed generally – the sentences mentioned belong to the O-language of a syntax. In that case a proposition is an O-sentence.[39] This

[37] Note that 'clause' is used in a sense slightly different from that of ordinary grammar. Perhaps the term 'nexus' of Jespersen [PGr] would be better; but it too does not fit exactly.

[38] This "mentioning" is that in the elementary statements of the theory – i.e. in the A-language.

[39] The question of just what an O-sentence is requires some discussion. In this paper 'sentence' is used in its ordinary grammatical sense, viz. as denoting an expression of a natural language (in particular the U-language) which has a certain kind of meaning (viz. it expresses a statement). Then, being a sentence is a contensive, not a syntactical, property of O-expressions; whether a wef is or is not a sentence is just as irrelevant for syntax as whether it gives any information about Plato's nose. (Cf. [CLg], § 1S2.) Thus 'O-sentence' can hardly mean anything else than 'sentence in the interpreted O-language'. For this purpose the word 'proposition' can equally well be used. In either case it is a fallacy to confuse the contensive term with a syntactical term like 'wef'. Of course, those who talk about the "sentential calculus" are not necessarily making this particular fallacy. Rather they are using for 'formula' a term which suggests the intended interpretation. This can be done without confusion if one is careful, and is often a help

is certainly an admissible interpretation of 'proposition'; but we are not committed to it any more than to the existence of esoteric entities. It has certain difficulties. Thus the custonary name for the sentence (4) would be the clause

'Anne is happy'

and one may hestitate to identify this with the clauses (5).

We may sum up this discussion as follows: We have four terms: 'statement', 'proposition', 'sentence', 'clause'. The first two of these we treat as nonlinguistic terms; the last two are linguistic, a clause being a special kind of noun. A clause is the name of a proposition; a sentence is the linguistic expression of a statement. Finally we can regard a proposition as the meaning of a statement in the sense that there is a correspondence between statements and propositions such that a statement asserts the corresponding proposition and a proposition is asserted by the corresponding statement. All of this can be understood without a commitment to a special kind of ontology. The terms 'proposition' and 'statement' can be construed in different ways all equally consistent with the above postulates; the choice between these ways is quite irrelevant.[40]

A further word is, however, in order in regard to the relation between the linguistic and nonlinguistic terms. For example, many persons will want to identify a sentence with a statement. That position is not wholly inconsistent with what has been said here, and a reader who prefers that view can easily make any necessary changes. However, I have used the term 'statement' where the emphasis is on content, 'sentence' where the emphasis is on linguistic character. Thus if we have two different presentations of the same formal system I should prefer the term 'sen-

to understanding. Even so, I find the variety of uses of 'sentence' rather confusing (cf. [LLA] pp. 85–87, or the review of Tarski [ILM]) and favor adopting, in the present sense, the otherwise useless term 'proposition'.

[40] It is characteristic of mathematicians that they deliberately try to make their theorems susceptible of as many different interpretations as possible. Certain features are insisted on; other considerations are ignored. The fact that the nonlinguistic terms are somewhat ambiguous is, from the mathematical standpoint, an advantage rather than the reverse.

tence' in discussing the system with reference to a particular A-language; 'statement' in discussing the formal system with the understandig that the particular A-language is irrelevant.[41] Admittedly there are intermediate cases where it is not clear which term should be used. Possibly further analysis will make the distinction more precise. If we had a precise formulation of with term should be used. Possibly further analysis will make the distinction more precise. If we had a precise formulation of synonymy we could say that synonymous sentences correspond to the same statement, and that 'statement' should be used when replacement of a sentence by a synonymous sentence would not affect the sense.[42] But it is not necessary to settle this question here.

5. *Interpretation of implication.* In the interpretation of propositional algebra we shall consider primarily interpretations in which the statement (1) corresponds to the proposition p in the sense of § 4. That is, if 'p' is replaced by any clause, (1) is a sentence which states that the proposition named by the clause is true.[43] We shall call such interpretations, and these only, *normal interpretations.*

To get a clear idea of the limitations imposed by a restriction to normal interpretations, we must recall that (1) holds only

[41] Thus even if one will not admit a semantical difference, one can admit a rhetorical one.

[42] We need terms at various levels of abstraction in ordinary discourse also. Thus the same object may be referred to according to circumstances as a fruit, an orange, or a Temple. In connection with sentences we may well need several such levels of abstraction.

[43] I.e., the proposition is in T. We have here a fixed interpretation. For a tautologous interpretation an analogous relation must hold for each substitution instance of p in an extension of H containing constant formulas. Since a tautologous interpretation thus reduces to fixed interpretation, we do not consider it further. (Note that a normal interpretation requires that $\supset$ be interpretable as an operation in H^*, – i.e. the interpreted H –, and this operation must be an implication in the sense of § 4; so that H^* is, in some sense, also a system. But the interpretations of the assertible formulas of H may not exhaust those which are true in H^*. Note also that it is not asserted that nonnormal interpretations may be interesting and important.)

when it is derived in the system H. Thus, in a normal interpretation, if p is a proposition, or in other words if 'p' is an abbreviation for some clause, then 'p' is not synonymous with 'that p is true' but rather with 'that p is assertible'. From this standpoint the usual interpretation of classical propositional algebra, and also Myhill's interpretation of (3), are both nonnormal. They can be made normal only if we conceive of the system H (or S of § 4) as being extended so that all true propositions are assertible in them.[44]

In § 3 we agreed that the symbol '$\supset$' is to be taken as an operator which combines two clauses to form another clause. We agreed further that the operation so designated is a proper implication only if the properties (a) and (b) hold. It follows from this that the meaning of any proper implication is fixed by that of (2) and the requirement that the interpretation be normal.

The various sorts of proper implication differ in the nature of (2). However in § 3 we saw that absolute implication is a minimal implication in the sense that it is a proper implication and that every elementary theorem true for it is true of any proper implication. Other implications can be defined by propositional algebra, with modus ponens as the sole rule, by adjoining further axioms to HA. Thus one gets a formulation HC for material implication by adjoining the axiom scheme

$$(6) \qquad ((p \supset q) \supset p) \supset p.$$

The study of the interpretation of implication should end at this point. But there are certain matters which, because of attention paid to them in the literature, require explicit discussion even if that involves some repetition.

In spite of the fact that (3) is a property of any proper implication, it still seems paradoxical to many persons. Let us there-

[44] Such a possibility is conceivable. The system H would not be a formal system in the strict sense, because the restrictions of § 1 would not be fulfilled in a constructive (finitist) sense. We can call such a system (following Carnap) an *indefinite* system. An interpretation in terms of indefinite systems can be admitted for cases where constructiveness is not important.

fore examine a special case. Let 'p' be an abbreviation for 'that Tuxie is black' where Tuxie is a certain black dog, and let 'q' be an abbreviation for 'that the moon is made of green cheese'. We may admit that p and q are propositions, and therefore admissible as atomic propositions in an interpreted propositional algebra. Then one argues, intuitively, that p is true, whereas, since p and q have no connection with one another, $q \supset p$ is false, thus contradicting (3). But in a normal interpretation the truth of p is equivalent to its assertibility in the system H. If H is not such as to include (1), then p is not true; if it is, then $q \supset p$ says simply that p is true in the system formed by adjoining q to H, which is obviously true. It does not make any difference whether the implication is strict or material so long as it is proper. There is no paradox unless one is using a different system H in talking about p from that used in talking about $q \supset p$, which is another way of saying the interpretation is nonnormal.

The fact that the interpretation of $p \supset q$, as well as that of p and q, depends on H, and that for a normal interpretation this system must be the same for all three, seems to have been overlooked by Ackermann.[45] Implication is indeed a relative concept. Its significance is, however, defined for any formal context (subject to the requirement of the normality of interpretation) by the rules (a) and (b).[46] This is made more explicit by the formulation in terms of Gentzen's L-rules. The terms 'absolute implication', and also 'material implication', have a significance apart from any context; the former is the minimal proper implication; the latter is the minimal one such that (6) holds.

The absoluteness of absolute implication does not depend on any claim to its being a definition of logical consequence. It does not pretend to be anything of the sort. It may very well be possible to conclude from (1) that

(7) $\qquad \vdash q$

---

[45] In his review of [TFD].

[46] In the case of material implication one must understand (6), or something equivalent to it, as part of the formal context.

without it being true that

$$\vdash p \supset q.$$

A simple case of this is where q is obtained from p by a substitution for certain atoms. In that case one does not adjoin (1) to the axioms and operate according to the rules; on the contrary one supposes a proof of (1) in H and shows that if the substitutions are made throughout this proof the result is a proof of (7).

Finally, it goes without saying that absolute implication is an objective concept in precisely the same sense that material implication is. It does not depend on any subjective feelings of "entailment". The deducibility which is associated with it is a deducibility by explicit rules, and not any kind of a priori deducibility of Platonistic character.

6. *Strict implication.* No discussion of the nature of implication would be complete without at least passing reference to the notion of strict implication. We turn here to the consideration of it. The term is used in two distinct senses in the literature. We consider these two senses separately.

The first sense of strict implication is an implication which is closer to actual deducibility than is material implication. If by deducibility one means formal deducibility with respect to explicitly stated rules, then it follows from the foregoing that absolute implication is the strictest of the strict. In terms of this kind of strict implication (3), as we have seen, is valid.

The second sense of strict implication is that of implication defined in terms of a notion of necessity (which may itself be defined in terms of other notions). If we indicate by a prefixed '#' a new operation on propositions, then one can define

$$p \rightarrow q \equiv \#(p \supset q). \tag{8}$$

If # is interpreted as necessity the implication so defined is a strict implication. This is the original sense of the term, as it was introduced by Lewis. Our discussion from now on will relate to strict implication in that sense.

In his investigations on this subject, Lewis proposed postulates for systems of propositional algebra in which $\rightarrow$ appeared as de-

fined notion. In the choice of these postulates he was apparently guided by his intuitive feelings. He was evidently seeking to define an implication (8) which was at the same time a strict implication in the first sense. The deducibility which he had in mind, however, was not formal, but was intrinsic and subjective. Consequently it may have seemed an advantage to define it in terms of necessity, a notion which, though also subjective, appeared somewhat simpler. But among the series of systems which he studied he was unable to decide which of them "expresses the acceptable principles of deduction".[47]

In the various systems evolved by Lewis the strict implication is improper. Indeed the statement scheme

$$(9) \qquad \vdash p \rightarrow (q \rightarrow p)$$

turns out to be not generally valid. In this sense Myhill's contention is correct.[48] On the other hand Gödel[49] showed that absolute implication (indeed the whole Heyting algebra) could be validly interpreted in terms of the Lewis system S4.

The notion of necessity, however, does not have to be conceived in the subjective fashion which characterizes the work of Lewis and his followers. It can be explained in objective terms as implication can. The first to do this was McKinsey.[50] It can also be done[51] in terms of formal deducibility by introducing the notion of deducibility with respect to part of a given system. Let

[47] Lewis, C. I. and Langford, C. H. [SLg], p. 502.

[48] In that case the objection to Myhill is that he appears to insist that this is the only possible interpretation. The "haverings" of which he complains are the result of this insistence.

[49] See Gödel [IIA]; also McKinsey and Tarski [TSC]. At the end of Gödel's paper there is an argument to the effect that an interpretation of necessity as provability is not acceptable in a system strong enough to include arithmetic. We are not considering a system of that strength here, but only propositional algebra. The original methods of Gentzen applied to propositional algebra and restricted predicate calculus only; the theory of [TFD] applied to extensions whose rules satisfy certain conditions, but not, apparently, to arithmetic. (Cf. [TFD], p. 64.) The matter can hardly be gone into further without technicalities.

[50] See his [SCS].

[51] See [TFD] Chapter V; also [ETM].

us call this part the *inner system*, while the whole system will be called the *outer system*. Then we can introduce conventions consistent with the idea that $\#p$ is the proposition which is assertible in the outer system if and only if p is assertible in the inner. There are various technical difficulties connected with carrying out this idea; these will not concern us here. Various kinds of strict implication can be defined in this way. If classical logic is valid in the inner system we are led to the Lewis system S4. One can also have absolute strict implication and still other varieties which have been only imperfectly studied.

These methods are capable of taking some of the mysticism out of the idea of necessity. If we give up the notion that there is an intrinsic deducibility, either strict or nonstrict, between propositions which is independent of any formal system, then there are several potential applications for the idea. It enables us to take into account cases where there are two levels of truth. Thus the inner system can represent theoretical truth of some kind, the outer contingent truth; the inner can represent logical truth, the outer physical truth; the inner intuitionistic mathematics, the outer classical mathematics, etc. One can even go on to have more than two stages, and to introduce considerations of possibility.

In the light of the foregoing we can now see why the strict implications are improper. For the interpretation of a strict implication requires two systems; in the case of (9) for example, p and q are referred, in the ordinary interpretation, to the outer system, whereas $\dashv$ refers to deducibility in the inner. In a reinterpretation in which only one system is involved (9) would be valid. This is the basis of Gödel's result.[52]

It is noteworthy that all the attempts to justify strict implication semantically lead to systems at least as strong as S4. Lewis himself expressed considerable doubt as to some of the principles of S4. But these doubts seem to have no semantic justification. The systems weaker than S4 may still be interesting; but the most important applications now in sight call for at least S4.

---

[52] See note 49.

Finally a comment is in order in regard to the incomplete character of investigations on modal logic. The studies of necessity above mentioned have an important limitation, viz. that the inner and outer systems have the same propositions and differ only in their axioms and elementary rules. The cases where they differ in their logic could be of interest for the application where the inner system is intuitionistic, the outer classical.[53] Again, the theory of possibility is in a quite unsatisfactory state.

BIBLIOGRAPHY

References to this Bibliography are by author and abbreviated title consisting of three letters in brackets. When no author is mentioned the author is Curry, or, in the case of [CLg], Curry and Feys. Reviews are listed under the work reviewed.

Braithwaite, R. B.

[SEx] *Scientific explanation. A study of the function of theory, probability and law in science.* Cambridge, England, 1953.

Church, A.

[IML_2] *Introduction to mathematical logic.* Second edition. Princeton, New Jersey, 1956. Review by Curry, *Journal of the Franklin Institute,* vol. 264, pp. 244–246 (1957).

[MiL] Minimal logic. (Abstract.) *Journal of Symbolic Logic,* vol. 16, p. 239 (1951).

[WTI] The weak theory of implication. *Kontrolliertes Denken, Untersuchungen zum Logikkalkül und zur Logik der Einzelwissenschaften,* edited by A. Menne, A. Wilhelmy, Helmut Angstl (Festgabe zum 60. Geburtstag von Prof. W. Britzelmayr), Munich 1951, pp. 22–37. Review by Curry and Craig, *Journal of Symbolic Logic,* vol. 18, pp. 177 f. (1953), and correction *ibid.* p. 326.

Curry, H. B.

[ALS] An analysis of logical substitution. *American Journal of Mathematics,* vol. 51, pp. 363–384 (1929).

[APM] Some aspects of the problem of mathematical rigor. *Bulletin of the American Mathematical Society,* vol. 47, pp. 221–241 (1941).

[CFS] *Calculuses and formal systems.* To be published in *Dialectica.*

[DFS] Definitions in formal systems. *Logique et Analyse* vol. 1, pp. 105–114.

[ETM] The elimination theorem when modality is present. *Journal of Symbolic Logic,* vol. 17, pp. 249–265 (1952).

[53] This case seems very plausible, but has not been checked in detail.

[GDT] Generalizations of the deduction theorem. *Proceedings of the International Congress of Mathematicians* 1954, pp. 399–400.

[LFS] Languages and formal systems. *Proceedings Xth International Congress of Philosophy* 1949, pp. 770–772.

[LLA] *Leçons de logique algébrique.* Paris and Louvain, 1952.

[LMF] Language, metalanguage, and formal system. *Philosophical Review,* vol. 59, pp. 346–353 (1950).

[PBP] Philosophische Bemerkungen zu einigen Problemen der mathematischen Logik. *Archiv für Philosophie,* vol. 4, pp. 147–156 (1951).

[TEx] Theory and experience. *Dialectica,* vol. 7, pp. 176–178 (1953).

[TFD] *A theory of formal deducibility.* Notre Dame Mathematical Lectures no. 6, Notre Dame, Indiana, 1950. Second edition 1958. Review by W. Ackermann, *Journal of Symbolic Logic,* vol. 16, pp. 56–58 (1951).

Curry, H. B. and Feys, R.

[CLg] *Combinatory logic,* vol. 1. Amsterdam, 1958.

Gentzen, G.

[ULS] Untersuchungen über das logische Schliessen. *Mathematische Zeitschrift,* vol. 39, pp. 176–210, 405–431 (1934).

Gödel, K.

[IIA] Eine Interpretation des intuitionistischen Aussagenkalküls, *Ergebnisse eines mathematischen Kolloquiums,* Heft 4, pp. 34–38 (1933).

Hilbert, D. and Ackermann, W.

[GZT] *Grundzüge der theoretischen Logik.* First edition. Berlin, 1928.

Jespersen, O.

[PGr] *The philosophy of grammar.* London, 1924, 1935.

Kleene, S. C.

[IMM] *Introduction to metamathematics.* Amsterdam and Groningen, 1952.

Lewis, C. I. and Langford, C. H.

[SLg] *Symbolic logic.* New York and London, 1932.

Lorenzen, P.

[EOL] *Einführung in die operative Logik und Mathematik.* Berlin-Göttingen-Heidelberg, 1955.

McKinsey, J. C. C.

[SCS] On the syntactical construction of systems of modal logic. *Journal of Symbolic Logic,* vol. 10, pp. 83–94.

McKinsey, J. C. C. and Tarski, A.

[TSC] Some theorems about the sentential calculi of Lewis and Heyting. *Journal of Symbolic Logic,* vol. 13, pp. 1–15 (1948).

Myhill, J.

[ISg] On the interpretation of the sign '$\supset$'. *Journal of Symbolic Logic,* vol. 18, pp. 60–62 (1953). Review by P. Bernays, *ibid.,* vol. 20, pp. 178–179 (1955).

Rosenbloom, P.

[EML] *The elements of mathematical logic.* New York, 1950.

Rosser, J. B.

[LMth] *Logic for mathematicians.* New York, 1953.

Tarski, A.

[BLF] Über den Begriff der logischen Folgerung. *Actes du Congres International de Philosophie Scientifique* 1935, vol. 7, pp. 1–11.

[ILM] *Introduction to logic and to the methodology of the deductive sciences. Translated by Olaf Helmer.* New York, 1941. (Appeared originally in Polish, 1936, and was translated into German, 1937.) Review by Curry, *Bulletin of the American Mathematical Society,* vol. 48, pp. 507–510 (1942).

The meanings of probability statements

by

SUNE HJORTH
(Sundsvall, Sweden)

Do probability statements concerning unseen events depend on logical implication? There are, in fact, logicians who maintain that this is so. As a consequence they are committed to considering the "justification of induction" as a meaningful problem, worthy of serious discussion. In order to throw light upon the matter let us choose as an example an utterance of a very common type: "All educated Englishmen I have met with so far have been nice people. Probably the rest of them are nice people too." Could it reasonably be said that the second statement is logically implied by the first?

It seems as if serious objections could be raised against this view. How could the second statement be deduced from the first one by virtue of conventional rules of language, if it is no tautology but contains new facts? Even if it were a tautology, it could not possibly at the same time refer to other events than those contained in the premiss. To say that one sentence "logically implies" the other in a case like this is a rather confusing terminology. The relation involved here is fundamentally of an empirical, not of a logical nature. To bring this out more clearly we may recast the utterance in another form: "The series of facts F_1, F_2 ... F_n makes me believe that the series will continue." In other words, one set of experiences *indicates* another, i.e. certain external stimuli have a certain effect on a persons mind, make him expect something. The utterance states a psychological fact, that is all there is about it.

If the view taken here is correct, the probability statement presented at the beginning is no logical conclusion, drawn from an assumed premiss. It is itself an assumption, a hypothesis, based on empirical evidence. Experience has taught us to rely on such

assumptions. If we did not, we simply could not exist as human beings. We certainly have no need for any "justification" of a non-empirical nature for making generalizations or assumptions about future events. Inductive "reasoning" should, as Ayer puts its, be left to act as judge in its own cause.

Now let us turn to another type of probability statement. If we assume that the relative frequency of black balls in a box is n %, and that this ball is a ball in the box, then, with a probability of n %, it is black. Here we have, clearly, a logical implication of the syllogistic type, though with a numerical value. For the usual quantifier "all" in the major premiss the quantifier "n %" is substituted, which value with logical necessity is transferred to the conclusion. We notice that the numerical probability statement is not used collectively, but *refers to a single case*. It may be regarded as a convenient means of transferring the value of a relative frequency to a single case.

Some logicians characterize this implication as "partial" or "inconclusive", saying that the premisses do not imply the conclusion with logical necessity, but only to a certain degree. The probability expression itself, they maintain, does not belong to the conclusion but is a meta-language addition to it. It will seem as if the holders of this view used the term "logical necessity" in a somewhat obscure manner, letting it mean something else than implication by virtue of conventional rules of language. Their restrictions as to the inclusion of the probability expression in the conclusion appear rather arbitrary.

Much discussion has been devoted to probability statements concerning coins or dice. If we assume that a physical body, say, a coin is so shaped that it falls as easily on one side as on the other, then we may conclude that, with a probability of ½, we will get tails when tossing it. This is evidently a purely logical implication based on an assumption about the possible behaviour of the coin. On the other hand, an empirical generalization is made, if, after getting n % tails in a long series of tosses, we say this makes it probable that the coin will behave in that way in future too, i.e. that the relative frequency of n % tails will persist approximately.

The examples with the coin lead us to the consideration of more complicated types of probability statements. The empirical generalizations so far considered have not expressed a numerical value. Sometimes, however, probability statements concerning future events are given the form of numerical statements, though, actually, they are still as much empirical generalizations as ever before. There are two typical instances of this, one more common, the other more technically refined.

After getting n % tails in a long series of tosses with a coin, we may say that there is n % probability that the next toss will give tails. Appropriately, it seems, we should say that *it is probable that, with a probability of n %, the next toss will give tails,* but the longer expression is shortened for the sake of convenience. As far as I know (and I may be mistaken) the possibility of this interpretation has been overlooked in discussions on probability. If this is so, one need not be surprised at the fact that no agreement as to the nature of probability statements has as yet been reached.

A statistician, however, is likely to choose another alternative. He might say that, with a probability of m %, the relative frequency of tails in a prolonged series might be expected to be n % $\pm$ d %. This complicated way of expressing things need not, however, lead us astray. The meaning is simply this: Assuming that our sample is only one of a theoretical distribution of possible samples, then only m % of these samples will have frequency values approximately the same as the true value of the population. There will thus be 100 – m % chances that the frequency value of our sample is more misleading than can be accepted.

The numerical expression of the probability in the latter case may be said to be on a higher level than in the former case. The first example refers to a single event without question. It is true that the second example refers to a series of events, but these are taken together as a single event, characterized as "sample", in relation to a still larger series of events, a distribution of such samples. Thus, once more we see that numerical statements, even in a subordinate position, are not used collectively but refer to single events.

If the analysis undertaken here is correct, probability statements

thus must be divided into two main groups: those which, by means of logical implication, transfer the numerical value of a relative frequency to a single case, and those which generalize empirical observations in the form of assumptions expected to come true. It seems to be a mistake to regard the latter category as conclusions in logical arguments. They actually will be no conclusions even if they are given the misleading form of numerical probability statements on single cases.

It follows, then, that the so called justification problem, which has haunted the world since the days of Hume, must be regarded as as pseudo-problem. Like other ghosts it simply does not exist, being a product of misuse of language. If empirical generalizations are no logical inferences at all, then there is nothing to be "justified". How safe our prediction of future events is, does not ultimately depend on our following logical rules for valid reasoning, but on our ability of reacting adequately on our experiences.

Do we know that basic norms cannot be true or false?

by

ARNE NÆSS
(Oslo)

Outline [1]

The question whether norms can or cannot be true or false is largely discussed as if it were capable of a solution *a priori*. In this paper I try to show that the traditional *a priori* arguments against the possibility that norms are true or false are not decisive and that *a posteriori* arguments may at least become relevant.

One of the main contentions in what follows is that if it can be the case that something ought to be, then it can be true that something ought to be, and that whether something is the case cannot be known *a priori*. Experience may disconfirm our hypotheses as to what is the case – if experience can disconfirm anything.

Another main contention is that in relation both to basic norms and to basic non-normative statements of science there are grave questions of justification which make the application of the terms 'true' and 'false' to them problematic. If the claim to truth-value of basic norms is as well (or badly) established as that of basic statements of science (and metascience), then the applicability of 'true' and 'false' to basic norms follows from their applicability to basic statements in the sciences.

---

[1] Some parts of this paper with slightly different conclusions, have been published as an article in *Logique et analyse*, Vol. 1, No. 1, 1958. Professor A. Ross emphatically disapproves of my suggestion in that article that value objectivism should be less icily considered than has been the case in recent Scandinavian philosophy of value. He refers to his own counter-arguments published in *Theoria*, Vol. 11, 1945. ("On the Logical Nature of Propositions of Value"). I have in what follows, especially p. 43 tried to show why his counter-arguments are not convincing. – Magister E. Stroheim has by his unpublished MS influenced my conclusions (towards apriorism).

A priori arguments

In *a posteriori* argumentation conclusions have an inherent *ad hoc* character: future experience, the term taken in wide senses, may overturn them. Or, if one retains the conclusion one may be forced to adopt increasingly complicated and arbitrary auxiliary hypotheses. The spirit of research on questions *a posteriori* shows itself in the systematic and careful way in which the researcher prepares himself for disconfirmation due to new observations. These may indeed overturn his *ad hoc* conclusions, but this does not impare his joy of *a posteriori* research. It is an essential part of the game.

In the debate concerning the objectivity of norms there is very little reference to contemporary or future empirical research or to other sources by which to improve upon the certainty or accuracy of the arguments. One reason for this may be that main arguments have a strictly *a priori* character, or are believed to have that character.

What are now the main arguments (if any) which have a justifiable claim of being *a priori*?

Let us first declare that the term '*a priori argument*' will be used to express a concept such that what ever will be *observed* in the future, an *a priori* argument will remain relevant (or irrelevant), true (or false). It cannot possibly change relevance- or truth-value because of such observation.

Such a concept of *a priori* does not have as conceptual characteristics generality, finality or apodicticity. The *possibility* of verification or confirmation, falsification or disconfirmation by future action is not denied. But *observational* statements will not be relevant.

The irrelevance of future observation refers only to observations concerned with the phenomena about which the argument asserts something.

The doctrine that norms cannot be true or false

The prevalent tendency in Anglo-American and Scandinavian philosophy and social science is to question or deny the *possibility*

of knowledge of a normative kind, for instance normative ethical knowledge. According to this dominant trend, we know or may know the shape of the earth, but we do not know and will not possibly come to know whether you should act so "dass die Maxime deines Willens jederzeit zugleich als Prinzip einer allgemeinen Gesetzgebung dienen könne". The difference is often expressed in condensed form by saying that whereas a proposition may be true or false a norm cannot be true or false. A norm may be valid or invalid, but that kind of validity is different from that of propositions, that is, different from truth.

Which are now the arguments put forth to strengthen the position that normative knowledge is impossible? In what follows we shall inspect a sample of proposed *a priori* reasons for the impossibility of basic norms being true or false. The sample includes an example of each of the main kinds or families of arguments put forth in contemporary philosophical debate in so far that they can be isolated from their specific, usually very complicated, context. The arguments which clearly are meant to be *a posteriori* are not included. Thus, the argument of A. Ross concerning lack of agreement on value-qualities of objects is left out.[2]

An argument from logic or semantics: norms are not propositions

Let us inspect the following argument:

True and false are, as used philosophically, predicated only of propositions. Norms are not propositions. Ergo, norms cannot be true or false.

Two counter-arguments

There are two counter-arguments to be mentioned at once. In framing *the first counter-argument* in what follows, we presuppose that the expression 'as used' means the same as 'as actually

[2] A. Ross, "On the Logical Nature of Propositions of Value", *Theoria*, vol. 11, 1945; a resumé is given in H. Ofstad, "Objectivity of Norms and Value-judgments according to recent Scandinavian Philosophy", *Philos. and Phen. Research*, vol. 12, 1951, p. 53 *et seq.*

used until now'. That is, the argument concerns the denotata, not connotation of truth and falsity.

If instances of the use of a predicate are known and arranged chronologically, we may at any definite time list common characteristics of the denotata. The next instance of use may however present a denotatum which lacks one or more of these characteristics. Even if it were the case that the denotata of the predicates 'true' and 'false' in philosophical literature always have been propositions, and norms are not propositions, this does not exclude the possibility that truth or falsity is predicated of a norm next time.

Some time ago 'carnivorous' in zoology expressed a predicate applied at that time only to animals. A materially adequate definition of 'carnivorous' might therefore at that time include a restriction to animals. The extension of the predicate included only animals. Today we speak also of some carnivorous plants. If somebody had eagerly proposed a definition such that the term 'carnivorous plant' is contradictory, *tant pis pour lui*.

It is not the business of philosophy to ossify terminological regularities. The argument that until now only propositions have been labelled 'true' or 'false', or, that certain definitions of yesterday include a restriction to propositions, beg the question. It presupposes that those beliefs are true which makes it convenient to adhere to a terminology that is under consideration. Now, this is a presupposition which may turn out to be false. If so, the definition of truth or of proposition may be changed with good reasons, reasons which are good because of certain changes in opinion, due e.g. to new discoveries. In short, from premisses telling us how the terms 'true', 'false', 'norm' and 'proposition' have been used until now, we cannot conclude that norms can, or cannot, be true or false. This holds whatever such premisses are telling us.

Now, if it were shown that norms might be true or false, the rules of ordinary logic could be applied to them. Norms derived from true basic ones plus hypotheses from various sciences would be true. Thus norms and (non-normative) propositions could be treated in the same way within important considerations. If such a possibility were realized it would furnish an argument for calling

the norms propositions, that is, a good argument for reconsidering an old classification, adjusting the delimitation of classes of phenomena to new insights, and changing terminology.

Summing up the first counter-argument, it may be thus formulated:

The fruitfullness or convenience of a classification system of entities into propositions and non-propositions depends on certain things we do with propositions, certain operations, for instance applying the calculus of propositions to them. If norms may be treated as propositions in making deductions, this might be a good reason for revision of the *fundamentum divisionis*: we might then include norms in a broad class of entities, the class of entities capable of being true or false, a class covering both non-normative propositions and what might be called normative propositions.

A second counter-argument is relevant if truth-value of norms is rejected on the ground that it violates rules of vocabulary or (nominal) definition: There are many proposals for definition of 'norm' and 'truth' none of which enjoys general consent among philosophers. But even if a now generally accepted definition ruled out the possibility of true or false norms, the terms 'true', 'false' and 'norm' have for a long time been used in such ways that to speak about norms as true or false has not involved contradiction. It is a perfectly legitimate undertaking to study the relation of truth and normativity on the basis of the non-contradictory usages, thus rejecting a definition that within a certain time interval was generally accepted. In short, since the terms 'true' and 'norm' have been, and still are, used in ways such that 'the norm N is true' will not constitute a contradiction, references to nominal definitions of such a wording that 'the norm N is true' is made into a contradiction in terms cannot show or prove that norms cannot be true or false.

We admit that 'norms cannot be true or false' is true in relation to all those systems of nominal definitions which make the sentence a *contradictio in adiecto*, but add that these are not all systems, and that the other ones are in no bad relation to common usage.

– Tentatively we conclude that the first argument is a bad

argument. It does not support an *a priori* rejection of there being true norms or genuine normative knowledge. It presupposes that the rejection has already been justified on some other basis.

A semantical argument: norm-sentences are meaningless, and therefore cannot be true or false

Basic norms or more generally, pure norms are expressive of feelings, they lack the symbol function characteristic of sentences expressing propositions. They may therefore be said to lack cognitive or theoretical meaning. Now, the predicates 'true' and 'false' apply only to cognitively meaningful statements. Their connotation or conceptual characteristics entail such a limitation. Therefore, basic norms cannot possibly be true or false.

Counter-argument

The first premiss involves two hypotheses in psychology or the social sciences: that norms are expressive of feelings and that they lack the symbol function. These are interesting working hypothesis, which perhaps may be strongly confirmed by future research. Actually, the first would be regarded as well confirmed by many psychologists. But it is obscure why the function of expressing feeling should have lack of a symbol function as a necessary consequence. The second hypothesis is independent of the first.

Anyhow, both hypotheses are inadequate as part of an *a priori* argument. If the premiss were established *a priori*, it might have been justified to use the strong expression 'cannot possibly be true or false' in the conclusion. But if the premiss in the future will only be strongly confirmed, the conclusion will only attain the status of a strongly confirmed hypothesis, useful only in *a posteriori* argumentation.

Ontological argument against normative knowledge

Let us then proceed from the semantical or analytical argument to an ontological one:

A true statement asserts of what is that it is and of what is not that it is not. In other words it asserts that what is the case *is* the case and what is not the case *is not* the case. Or, truth is *adequatio rei et intellectus*, – truth consists in agreement with reality.

These formulae (and others) are traditionally used in references to the so-called classical conception of truth.

Now, says the argument, take a norm stating that this or that *ought* to be. Be and ought to be are not the same, neither are *not to be* and *ought not to be*. The Aristotelian tradition thus implies that the norm cannot be true or false.

Moreover, the formula *adequatio rei et intellectus* presupposes *res*, that is, an object which our statements says something about. This object must actually be there, not only ought to be there. Similarly the formula 'agreement with reality' presupposes a reality or state of affairs about which we are saying something.

Now, according to the Aristotelian tradition, there do not exist any entities either immanent in the world or transcending it, which are capable of forming the truth condition for a norm.

Counter-argument against the ontological argument

Let us inspect the sentences 'It is true that there are black swans' and 'It is true that you ought to love your neighbour as yourself'. The classical, Aristotelian trend in definitions of truth admits the formula that it *is true that* there are black swans if it *is the case that* there are black swans. What is the object, the res, the reality, with which the statement must agree in order to be true? It is the condition of something being so and not otherwise, that is, of something being the case. I think this answer does not amount to more than a repetition of the use of the expression 'is the case that'. The introduction of terms like 'object', 'reality' suggests extension. Perhaps something may be the case without presupposing anything extended. If this possibility is rejected in advance, the discussion of this article is made pointless by an *a priori* postulate.

Semantically the formula 'if it is the case that x, then it is true that x' applies to the Golden Rule. We may say that it *is true that*

you ought to love your neighbour as yourself, if it *is the case that* you ought to love him as hard as that.[3] If this is so, the norm asserts of what is that it is. It does not say that actually you love your neighbour. The state of affairs, or reality, it designates is the one *that you ought* to love your neighbour. Compare the following way of speaking: "*Is* it *so* that you ought to love your neighbour? – Yes, it *is so*". If this 'is' is too abstract, unsubstanstial, shadowy to permit the terms 'real' or 'state of affairs', let us drop those terms in this connection. 'True' and 'is the case' are good enough.

Correspondingly, a norm saying that you ought *not* to love your neighbour less than yourself, states of what is not so that it is not so. The Aristotelian definition of falsity applies. And so does the 'is the case'-formula. It is not the case that you ought to love him less than yourself.

The Aristotelian and other classical truth-formulae leads us to assert that at least some norms are true, provided *it is the case* that we ought to act as the norms "tell us". Whether it is the case can hardly be decided by inspection of classical formulae of truth.

Among the kinds of doctrines traditionally used to support the claim that it may be the case and may be true, that something ought to be, one main kind is outstanding because of its long and distinguished history: the doctrines of ideas in some more or less platonic versions. Applied to pure mathematics and logic we get the doctrines of mathematical objects such as (non-physical) circles or relations, classes etc. as entities distinct from the relata and the class-members taken separately.

Applied to norms, the doctrines of ideas may assert the existence of ought-relations or *'ideale Forderungen'*, not further reducible. Among sociologists, Georg Simmel takes this view.[4]

---

[3] Empirical tests of various kinds have shown that this way of speaking is rather common, but not, or at least not always, associated with an explicit or implicit belief in a normative or ideal "reality", a platonic heaven.

[4] "Nun gibt diese ideelle Forderung, die weder in einer objektiven Realität, noch in unserm Subjekt ihre Heimat hat, ebendamit ein Problem auf, dass man wahrscheinlich nur durch das Axiom lösen kann, dass diese als Anspruch des

The phrases 'world of value', 'world of duties', 'world of norms' suggest platonic objects in a normative heaven and belief in such objects requires an imagination more religious than scientific. The belief in the possibility of normative knowledge does not require this imagination. It is enough, e.g., to believe in the possibility of a kind of insight which Husserl has described in his account of ideal, apodictic laws. He refers to the principle of identity and analogous first principles which he believes can be *seen* to be valid by a confrontation *with what is the case*, with the "Sachverhalt selbst". I cannot say that I find Husserl at all convincing, but it seems to me not unreasonable to suppose that there might be instances of pure intuitions of the phenomenological kind.[5]

If there were such intuitions, they would establish normative knowledge. In Husserl's "meeting" with what is the case, there is no schism between a physical or perceived object and a mind, but an illuminating insight in which it is seen *that* something is the case. It is not the *something* but the *that* which is illuminated.

According to platonism the sentence 'It is the case that the center of a circle halves all its diameters' may be a true or false as a statement about an ideal circle. According to Husserlian doctrines, it would rather be the logical relations between the definition of a circle and that which is asserted by the sentence that can be intuited. There is, strictly speaking, no need of ideal *circles* according to such doctrines. The ideal state of affairs do not require an object, a circle or something else, which exhibits the state. The position that some norms possibly are true may thus be given a Husserlian colour. 'It is the case that you ought to realize yourself' might be a true or false statement about an ideal state of affairs of a normative kind.

Daseins an uns auftretende Ordnung eine selbständige, nicht auf Bekannteres zu reduzierende, völlig autochtone Kategorie ist." "Es kann diese Entgegengesetztheit der Ursprünge zu haben scheinen, weil es tatsächlich von keinem dieser herkommt, sondern einen ebenso primären und eigenrechtlichen Ursprung hat, wie das subjektive Leben und die äussere oder die geschichtliche Realität." *Hauptprobleme der Philosophie*, 7. ed., p. 117.

[5] A critique is attempted in my "Husserl on The Apodictic evidence of ideal laws", *Theoria*, vol. 20, 1954.

Among the contemporary advocates of platonic varieties of objectivists, Nicolai Hartmann might be mentioned. A couple of quotations will exhibit his way of thinking: "Ethik kann tatsächlich lehren, was sittlich gut ist, wie Geometrie lehren kann, was geometrisch wahr ist. Aber sie kann dem sittlichen Bewusstsein nichts aufdrängen, sondern es nur auf seine eigenen Inhalte und Prinzipien hinlenken. Sie kann nur aus ihm heraufholen, was in ihm enthalten ist. Auch hierin gleicht sie der reinen Mathematik. Der Unterschied ist nur, dass die Prinzipien und Inhalte, die sie ins Bewusstsein hebt, Gebote, Normen, Werte sind. Dem Inhalt nach also ist sie normativ, nicht aber der Methode oder der Art der "Lehre" nach. Denn die Apriorität der Einsicht und die Didaktik der Hinführung auf sie ist die gleiche hier wie dort." "Die ganze Verantwortung für Rechtmässigkeit und Objektivität des Wertungsmasstabes fällt der eigentlich apriorischen Wertschau, d.h. letztlich dem Wertgefühl selbst zu." [6]

"Das reale Sein also kann wohl Wertvolles enthalten, aber es kann kein Sollen enthalten. Es kann auch das Hintendieren als solches nicht aufnehmen, wenn nicht eines seiner eigenen Gebilde sich als tendenzfähig d.h. als aufnahmefähig für das Sollen erweist. Das Subjekt ist das Tendenzfähige – und so weit wir wissen können, das einzige Tendenzfähige – der realen Welt. Es allein kann das Sollen ins Sein „setzen".

So kommt es, dass das Sollen, obgleich es nicht im Subjekt wurzelt, ihm vielmehr als gestellte Anforderung gegenübertritt, dennoch als aktuale Tendenz im realen Sein nur am Subjekt ansetzen und nur von ihm aus von Reales determinieren kann." [7]

The chief difficulty in refuting *a priori* the contemporary platonic or objectivistic views of values and norms lays therein that those views affirm the *existence* of something. In matters of ghosts, golden mountains, talking stones and many other entities defined without internal contradictions, refutations are based on experience or, more generally, on *a posteriori* considerations of some sort. The refutations may be of crushing weight, making be-

[6] N. Hartmann, *Ethik*, Berl. 1949, p. 29, 127.
[7] N. Hartmann, *Ethik*, Berl. 1949, p. 182.

lief in the existence of the entities practically impossible. But they are not *a priori* refutations. 'Think away' your experience until now, or imagine you had had other experiences, for instance those described by Husserl, other observational journals, and your refutation would not hold any longer. One may be convinced never to face such observational journals, but it seems difficult to find valid *a priori* justifications of this conviction.

If a norm is true is it then also a description of something?

Affirming the thesis that for all we know *it may be the case* that something ought to be, do we not also assert that, for all we know, normative propositions may be a subclass of descriptive?

If one takes the view that all sentences of the kind 'It is the case that A' are descriptive and 'B ought to be' is normative only in so far as it is *not* synonymous with 'It is the case that B ought to be', then we may answer affirmatively. But it would not alter the important conclusion that those sentences of the kind 'B ought to be' which are synonymous with 'It is the case that B ought to be' may be true or false. The proposed terminology would lead us to subsume an important class of ought-sentences under the concept of descriptive sentences. And why not?

Because of the importance of concepts of 'norm' in debates on value-nihilism, it may be worth while to discuss an attempt by H. Ofstad to give a descriptive definition of 'pure norm' adapted to the use it has got in the writings of the Uppsala philosophers.

In his article already referred to on Scandinavian Philosophy (p. 46) H. Ofstad defines 'pure norm' in such a way that it excludes any norm which *may* be expressed by a declarative (descriptive) sentence:

"A formulation given by a person is said to express a pure norm (for this person (insertion by A.N.)) if and only if the following two conditions are fulfilled:

1) the formulation must be synonymous for its giver with an imperative formulation, or with a formulation which says or denies that something ought to be done, or shall be done, or must

be done, or that something has value or is good, or is bad, or is better than something else or the best of something.

2) the formulation must not be synonymous for its giver with a descriptive (declarative) sentence."

The terms 'declarative' and 'descriptive' admit different interpretations of importance for our present discussion: if 'declarative' is defined as is customary in grammars, sentences like 'It is your duty to defend your fatherland', 'You ought to abstain from killing any human being', are declarative. Thus, all sentences fulfilling the first condition might turn out to violate the second condition with the exception of the imperatives.

If on the other hand, 'declarative' and 'descriptive' is taken from terminological trends in value-theory, the terms might as well be introduced by inserting a series of negations in 1): the formulations must not be synonymous with a formulation which neither says nor denies *that* something ought to be done, nor that something shall be done etc. Unhappily, there are no suitable definition of 'descriptive' except in contrast to 'normative' (or 'imperative').

It should be unnecessary to add that the term 'pure norm' thus introduced is only precise enough for very modest purposes of exposition. The attempts to introduce more precise and at the same time fruitful concepts of 'pure norm' or 'genuine value judgment' etc. have been, so far I can judge, rather unsuccessful.

Now, how is 'It is the case that B ought to be' to be classified? Empirical studies show that for many people in many situations the sentence is synonymous, or near synonymous with 'B ought to be', expressing forcefully the intended objective validity of the norm. Using the descriptive definition of Ofstad, we would unhesitatingly say that the sentence fulfill the first condition. But how should we tackle the question whether it is synonymous with a declarative sentence or not? In the absence of any suitable definition of 'declarative', we might say that the first part of the sentence *sounds* declarative, the second not.

If a declarative sentence is defined as a sentence expressing that something is the case, the sentence 'It is the case that B ought to be' is declarative. For those who uses that sentence synonym-

ously with 'B ought to be', this latter sentence will also be declarative. For people who use such a sentence as indicated by Stevenson and others, it would not be declarative.

How strongly should normative insights motivate action?

One of the main conclusions of Alf Ross concerning propositions of value is that they cannot be true or false and that if they could, they would not have a normative function other than that which declarative sentences sometimes do have.

Even if there were objective values, says Ross, "the *apprehension* of value would not be 'practical' or 'normative' in any other sense than all cognition is. For any cognition can be called 'practical' or 'normative' in the sense that on the assumption of a given aim of aspiration the insight into a certain theoretical state of affairs has a normative effect on behaviour. Thus, for instance, the theoretical insight that water boils at 100° C. will determine the behaviour of the person who wishes to make the water boil. The theoretical insight may be transformed into a hypothetical norm (in the present case: if you wish to bring water to the boil, you must heat it to 100° C.). It may also be expressed thus: *under suitable circumstances all theoretical insight possesses technico-normative significance.*

And the apprehension of value would have no other normative significance. The objective values, to the person not interested at the outset, would practically be a matter of complete indifference. How do these values concern my practical attitude? I may for instance accept the theoretical assertion that lying has the quality of 'odiousness' and at the same time without any sort of inconsistence lie on a large scale."

It seems here that Alf Ross works with a concept of 'normative' such that a sentence "Lying is odious" only would have a normative function and express a true norm, if insight into its truth would prevent a person from lying, or if it agitates the speaker or listener. About a preachers sermon it may, however, in all sincerity be asserted that it was true what he said, but it was boring and left the listeners indifferent. We may also sometimes say:

"I know what I now *ought* to do, but I do not *want* to do it." This and other similar utterances of occasional practical indifference towards what is sincerely believed to be true, cannot easily be taken as conclusive argument against the beliefs. Value objectivists stress the intense prescriptive character ("Forderungscharacter") associated with genuine acts of value cognition, but not to the extent that the subject is by necessity permanently and strongly motivated to satisfy the demands associated with the cognitions.[8] Thus, I do not think the argument of Ross hits the mark. Knowledge of values could be of great importance in practice even if it did not invariable elicit a behaviour in accordance with the insight.

A priorism incompatible with empiricism

In the light of the foregoing, my tentative position will be that *for all we know* some norms may be true or false and that consequently normative knowledge may in principle be obtainable, but perhaps never obtained.

But why bother with the possibility of such knowledge if one happens to be more or less *convinced that there is none*?

In my own case I bother with this possibility because I am not convinced that man might not have had, or might not in the future acquire normative knowledge. He will perhaps be able to verify or falsify a norm, or at least confirm or disconfirm it to a degree of practical importance. I am neither certain that there are normative truths to be verified or confirmed, nor sure that there are not.

[8] Cf. for instance what Henry Sidgwick, the intuitionist, has to say: "when I speak of the cognition or judgment that 'X ought to be done' – in the stricter ethical sense of the term ought – as a 'dictate' or 'precept' of reason to the persons to whom it relates, I imply that in rational beings as such this cognition gives an impulse or motive to action: though in human beings, of course, this is only one motive among others which are liable to conflict with it, and is not always – perhaps not usually – a predominant motive." (*The Methods of Ethics*, Book I, Chapter 3, § 3). "Unser Bewusstsein empfindet Forderungen an sich gerichtet, die es durch den Willen realisieren kann." (Georg Simmel, *Hauptprobleme der Philosophie*, Göschen Ausgabe, p. 114). It *can* be realized, but only if the prescription furnish a sufficiently strong motive.

The climate in Anglo-American and Scandinavian analytical philosophy seems to have been favourable to *a priori* arguments against the objectivity or truth of norms or values. But those who are inclined towards empirical trends in philosophy cannot but view with suspicion the dominance of apriorism, and this suspicion will concern counter-arguments as strongly as pro-arguments. It is the particular job of empiricism to leave questions open if there are no decisive arguments at hand.

Alf Ross seems to view his enquiry into the nature of value as a crusade against a priorism. But of what kinds are his own arguments? Neither mathematical nor (formal) logic. He calls his method "logical analysis". But from which kinds of enquiry research does this method gets its authority? It seems to me that in combatting a priorism in one field he introduces it in another. He does not specify any empirical research that might settle questions *a posteriori*.

Must normative objectivism lead to fanaticism?

The following is not intended to be a continuation of the argument. It states a causal hypothesis concerning conditions which have undermined objectivism. It has been felt that dogmatism and fanaticism in religion and morals, and absolutism and totalitarianism in politics have to a high degree been supported by the belief that *I* am or *my party* is absolutely right, and right in an absolute sense, as a fact, and beyond doubt. Objectivism may foster inhuman attitudes between human beings.

The belief in the actuality of insight, knowledge, true intuition, in morals, religious normative dogmas, ideological superstructures, has, according to the Swedish and Danish value-nihilists (Hägerström, Phalén, Alf Ross, to mention some whose works are in part available in world-languages), contributed immensly to the mental and physical violence so prominent in the history of European countries in the last centuries.[9]

---

[9] A. Ross, *Om Ret og Retfærdighed*, Kbh. 1953, p. 359. V. Lundstedt, *Superstition or Rationality in Action for Peace*, 1925.

I venture to suggest that the idea of normative knowledge would appear less repulsive if it could be combined with another idea that made its use in support of religious, moral, and political fanaticism unlikely or impossible. This idea is that of *difficulties of verification* as an obstacle to an easy switch *from truth to knowledge*, from possibility to actuality.

Let us take the Golden Rule as an instance. The belief that it *either is true* that you ought not to do against others what you do not wish others to do against you; *or that it is false*, and the belief *that it is true*, do not together make the Golden Rule a piece of knowledge. From the truth that Fermat's theorem either is true or false, plus a belief that it is true, it does not follow that the theorem is a piece of knowledge. Perhaps it will never be known. In those cases in which verification is difficult or practically impossible we are not entitled to proceed from the knowledge that p or non-p is true to the knowledge that p is true or that non-p is true.

And even in cases in which verification is not too difficult, we do not believe dogmatically and firmly in all our physical and mathematical beliefs as a consequence of our belief in an objective standard of mathematical and physical knowledge. We conceive that there may be a gap, wide or narrow, between our own beliefs and ultimate knowledge.

Applied to norms this suggests that the road from knowledge that *you are right or I am right* to the knowledge that *I am right* might be as difficult or even more difficult to traverse than the corresponding one related to non-normative knowledge. There may be practically unattainable requirements of ethical status, power of meditation, depth of understanding, integration of personality, freedom from prejudice, intensity of action and engagement, serene disinterestedness.

In suggesting aversion against absolutism and dogmatism as a source of negative attitude towards value objectivists, an "unscholarly" motive may be said to be imputed to the value-nihilists. Alf Ross suggests that such a motive operates in the camp of value objectivists: "What renders difficult the clarification of the problems to be discussed here is not only their inherent difficulty

but also the unscholarly motives that, rising from the depths of the soul, make so many cling to a spiritualistic metaphysic and the edifying belief in a moral spiritual order of the world. Hence what we require to attain clarity is not only the intellectual ability to master onerous tasks, but also a certain quality of character: a consistent scientific purity, liberation from every craving for a metaphysical belief which at bottom is rooted in impotence and fear." (P. 172.)

As regards the "belief in a moral spiritual order of the world" it is of importance to note that only a subclass of such beliefs are beliefs in a moral spiritual order *in* the extended world. The ontology of Hägerström, and perhaps also that of Alf Ross, is one according to which what is the case must be the case *in* a spatio-temporal world.[10] The more refined value objectivists do not presuppose a narrow concept of what is the case, and are therefore more difficult to argue against. What is the case may not always be the case somewhere. It is likely that an argumentation purported to be thoroughly "scholarly" (and therefore in the spirit of research) would have a more agnostic or sceptical conclusion than that of strong *negations* (It is *not* so that there is a moral order, it is *not* so that any norms are true etc.).

Disagreement about norms and how to verify norms suggest that verification is highly problematic

There is not sufficient ground for rejecting the possibility of norms being true, but sufficient ground why we should reject the assertion that somebody actually at the moment possesses normative knowledge.

The main reason is, I think, the persistent disagreement among presumably normal, competent people who make an honest effort to understand each other's ethical and other normative positions.

[10] The narrowness of this conception is well argued by Marc-Wogau in his "Axel Hägerströms verklighetsteori" (Tiden, vol. 32, 1940). The author thinks that scepticism as regards the existence of non-material worlds is sometimes due to lack of *empirical* reasons. This is in line with our own thinking.

The disagreement seems to be less prominent in relation to fundamental norms than in relation to derived ones. It should also be noted that in political and other ideological structures violent disagreement about what actually happens in the world plays a dominant part. Nevertheless there is in our time sufficient disagreement concerning norms to reject them as expressions of knowledge.

A similar negative conclusion is warranted in relation to the many proposed ways of verifying norms, whether intuitional or otherwise. As to the expositions showing difficulties of verifying value qualities ("tertiary qualities") in a way similar to secondary qualities, the reader is referred to the article of A. Ross already mentioned.[11] There seems at present to be no way of verifying or confirming a basic norm which can compare with the methods of the sciences, even if we include such shaky fields as parapsychology.

On the other hand, basic presuppositions of the established sciences are in a similar position as that of basic norms. But however chaotic the disagreement concerning ultimate foundations, many derived propositions and rules command near universal adherence. If q is a consequence of p, and q is strongly confirmed, this cannot but make us trust p at least tentatively, even if the way of confirming q itself rests on the supposition of p being true. This may in part explain any lack of verification and established knowledge of basic non-normative assumptions are not taken as a sufficient reason for general scepticism.

A planet with Hume's argument inverted

"In every system of mortality which I have hitherto met with, I have always remarked that the author proceeds for some time in

[11] One main source of the argumentation by Alf Ross in favour of value-nihilism is his requirement of truths that they should be testable or confirmable in a rather definite way described by A. Ross in detail. If this requirement is accepted as a basic postulate, the position of value-objectivists is indeed earnestly in mortal danger. But why should the objectivists accept it? It has a form of a postulate, and arguments presuming acceptance of the postulate are irrelevant to those who do not accept it. For criticism of the postulate see the quoted article of H. Ofstad.

the ordinary way of reasoning, and establishes the being of God, or makes observations concerning human affairs; when of a sudden I am surprised to find, that instead of the usual copulations of propositions, is, and is not, I meet with no proposition that is not connected with an ought, or an ought not. This change is imperceptible; but is, however, of the last consequence. For as this ought, or ought not, expresses some new relation or affirmation, it is necessary that it could be observed and explained; and at the same time that a reason could be given for what seems altogether inconceivable, how this new relation can be a deduction from others, which are entirely different from it." [12]

This famous passage introduces Hume's argumentation against rationalistic conceptions of norms. It presupposes a doctrine about what kind of reasoning is ordinary and why the ordinary should have any authority with us. If we start from a negation of that doctrine, the passage looses its force.

Let us suppose that there is a planet where physical laws are tremendously complicated, practically inpenetrable, and suppose the morally intensively interested and physically uninterested inhabitants agree completely on all fundamental norms. These might then get the status of self-evident axioms, and be contested only by cranks. There would be no embarrasing question of verification. Verification would only be required of derived statements and especially of norms derived in part by means of physical hypotheses.

On this planet where Dostoevsky and other perhaps would have felt more at home, a David Hume might have stood up and warned his fellow philosophers. How did they manage to proceed from safe normative propositions to the highly doubtful physical ones? Scarcely by ordinary reasons! Physicalizing authors forget, he would say, to tell how they obtain their physics from a set of normative premisses.

On this planet there would be physical nihilists proclaiming that physical statements are neither true nor false, and moreover, for *a priori* reasons, that they cannot possibly be true or false. The

[12] David Hume, *Treatise of Human Nature,* III, 1, 1.

world of physical laws is an illusion, they might say, and the world of common sense, that is, that of values, is the only reality. In order to be true, a proposition must agree with reality. But there is no reality to agree with for a statement in the systems of physics.

A planet with no argument

Conditions on the imagined planet and on our own have in common a disparity between the extent and intensity of agreement on norms versus that of non-normative propositions. Now, let us imagine a third planet, where *our* conditions relative to non-normative, especially physical propositions, and the conditions of planet No. 2 relative to norms, are realized. Will there be any place for a Hume there? There would presumably be free passage from non-normative to normative and *vice versa*, all in complete harmony with "natural reason", and conditions of verification or confirmation being agreed upon to the pleasure of both moralists and naturalists.

Basic norms and basic non-normative assumptions of science

It has often been pointed out that the methodology of testing the validity of basic norms lacks a chapter corresponding to that of observation in the methodology of non-normative statements of science. The comparison is unimportant, because the chapter on observation does not touch upon the foundations of science. The status of observation as a source of knowledge cannot itself be justified by observation. The psychology and physiology of sense-perception is itself based partly on observation and cannot be used for a justification of observation epistemologically, without a *circulus in probando*. The status of observation as a source of knowledge seems in science to be postulated or to be taken as intuitively evident. That is, the justification of this status does not seem to be different from the kind of justification needed in relation to norms, provided we have all found certain norms intuitively evident.

Comparing basic norms with basic non-normative statements relied upon in science it is seen that they have much in common from the point of view of methodology of testing: on the whole, "intuitions" are relied upon.

If the basic methodological rules of science are formulated as norms – as they often are – the difficulties of testing will be similar to those in science. The question arises whether a proposition in science arrived at by means of such a rule should be called true or false, if basic norms are denied truth value.

As regards the principle of induction, it is agreed that it cannot be justified by observation. Max Black and others prefer to speak of the principle as a policy, the policy of expecting certain things to happen rather than their opposites.[13] It is often formulated as a norm. If this norm is negated and a very different one adopted, the content in textbooks in physics or history would have to be radically changed.

All this adds nothing to the reason for believing in the possibility of normative knowledge, but eliminate certain misconceptions about non-normative knowledge. It has also its problems or even paradoxes. Its basis is obscure.

The foregoing seems to support the contention that the distinction between norms and non-norms in relation to truth-values is something rather upheld and justified by belief in what is or is not, that is, in what is the case, than by valid *a priori* reasons. In strange worlds into which our world may develop, for all we know, conditions may favour other beliefs. They are at the mercy of future experience.

In using the term 'belief' I do not intend to convey that the change in standpoint towards norms and non-norms would be irrational. The change may be justified by induction or any other kind of rational argument *a posteriori* which commends universal assent today.

Consensus omnium should not be taken, it is often said, as a reliable criterion of truth. But on the whole, what actually is con-

---

[13] Max Black, "Pragmatic Justification of Induction", in *Problems of Analysis*, Lnd. 1954.

sidered to constitute knowledge among researchers in a field is just those beliefs which are universally held by those who are considered to be experts. If there is much disagreement among experts about a proposition, none are considered (safe) knowledge. Practical variation of conditions of agreement, whatever their causes, changes our conception of what is real or objective, and the change may be in favour of normative or mixed normative and descriptive models or reality, away from the physical or other purely non-normative models favoured today among scientists.

A posteriori questions leading us to empirical research

Our conclusion from the foregoing is that arguments functioning in the debates concerning norms and truth-values as if they were *a priori* arguments, in part are not *a priori*, in part are *a priori*, but not decisive.

It is not the aim of this article to enlarge upon the *a posteriori* arguments. We should like, however, to emphasize that in many *a posteriori* considerations of general and abstract kinds, empirical research has been fruitful. It is methodologically sound to look for possibilities of such research relating to norms.

A task for empirical research of a non-semantical kind is, just to take an example, to find out how scientists and others *actually* justify their assertions (whether normative or not) when confronted with repeated persistent questions "why?" and "how?". Preliminary experiments suggest that the chains or arguments do *not* tend, in the long run, to end up with observation sentences.[14] It seems that methods of verification by non-philosophers generally comprise both norms and descriptions as links in the argumentation chains. This has some bearing upon the contention that there are no methods of verification of pure norms. There is no indication that the norms occurring in the links are all instrumental or otherwise different from basic or pure norms.

Another task of empirical research it is to study fluctuations

[14] A. Næss, "Gesetzmessigkeiten bei Argumentationsketten", *Erkenntnis*, vol. 7, p. 384.

in the conceptions of reality, especially the fluctuations in regard to the prestige of a space-time manifold as typical of what is real. Such studies would be relevant for our estimation of the sources of our intuitive certainty that it cannot be the case that something ought to be. If what is the case is always something in space and time, as, for instance, Hägerström seems to presuppose, it will never be the case that something ought to be. The strong conviction that value-nihilism is true may partly stem from belief in a kind of physicalist reality.

This does no more than hint at the kinds of *a posteriori* arguments which lend themselves to empirical research with more or less established methods or which ought to be studied in order to look for possibilities of empirical avenues of attack. There seems to be enough to do in this field for generations of researchers who are interested in philosophical reflections with empirical bearing.

Ultimate epoché

But will empirical research give decisive results? Or, more generally, can *a posteriori* arguments be decisive? It is difficult to see how any set of arguments what so ever can be decisive without already assuming rules of inference and basic descriptive assumptions. The "choice" at this initial stage of any argumentation will largely determine the force of empirical findings.

Thus the above reasonings result in both rejecting the decisive force of *a priori* and of *a posteriori* argumentation in answering the question of truth-value of basic norms. I cannot see any way leading to a rational decision. A suspension of judgment, an epoché, seems appropriate.

DISCUSSIONS

Past and present. Some remarks on A. I. Ayer's theory on past and present in his "The Problem of Knowledge". By *L. E. Palmieri.* (Chicago. Ill.)

After he surveys several views for ways of certifying memory and, in one sense, knowledge of the past, Ayer offers as his solution the suggestion that temporal precedence is given in experience. He writes:

> As a matter of empirical fact, one can hear or see A–following–B, in the same immediate fashion as one can see A–to the left of–B. And this relation of temporal precedence, coupled with the notion of the present, which can be defined ostensively, is all that is required to yield the concepts both of the past and of the future. Defining the present as the class of events which are contemporaneous with *this,* where *this* is any event that one chooses to indicate at the moment, one can define the past as the class of events which are earlier than the present, and the future as the class of events which are later than the present.[1]

Though Ayer has not defined 'following' nor written of the duration of A, and that of B, and whether the cut (the 'following' as it were) has a duration, I believe we can take it that he would not allow an extended duration from A through B. One thinks of the specious present and the use to which the notion has been put in attempts to explain matters similar to those which confront Ayer at this time, though I only refer to the notion hoping it presents some gratuitous sense of familiarity with what Ayer believes we see or hear with respect to A and to B.

The important question is whether Ayer is claiming that in this experience of A–following–B we have, literally, an experience of precedence in a temporal sense or not. If we don't then, while Ayer might use ostensive definition of the present, he can not proceed to define the past and future as he has proposed. And the would then seem

[1] A. J. Ayer, *The Problem of Knowledge,* New York, St. Martin's Press, 1956, p. 170.

to have failed to provide an acceptable substitute for the phenomenalist's view on the matter: a view which he now holds suspect. If, on the other hand, we *do* have such an experience of precedence in a temporal sense with, as Ayer assures us, the immediacy of A–to the left of–B, we would seem to have need for care that the chosen *this* of the given moment is not befouled with the past. But further thought reveals that the *this* is never free of the past, and the objection holds no matter how narrowly we define the duration of the specious present, or, to restrict ourselves to Ayer's term, however, we limit the duration of A–following–B.

A word or two of elucidation might be desirable here. All of us have an idea of the present. Though this might vary under examination, the notion varies between two limits: from a duration all parts of which are parts of an indiscriminate *now* to, on the other extreme, a dimensionless now, a cut, separating the past from the future. The former is sometimes referred to as the specious present, for though every part is accepted and even experienced as *now* yet we are told and there is some reason for believing that it is in part a false present held together by recall if not also anticipation. In any case the duration is allowed to be experienced as present. If Ayer would now claim that we directly, without memory and without inference, experience A–following–B temporally, then if the claim is not contradictory the *this* can not be used to define the present alone.

It might be objected (nay, has been, though in a private communication) that Ayer need only substitute 'present or immediate past' where he has written 'present', then with the relation of temporal precedence given in experience he can distinguish past from present, leaving a so-called residual problem of sorting present from immediate past. I should like to point out that if past is to be directly distinguished from present (which now is to be understood as 'present or immediate past') then 1) the distinction between present and immediate past would have to be made on a physical time distinction, which would be philosophically objectionable, or 2) the so-called residual problem is the very shadow of the original problem of distinguishing past from present, compounded by the new problem of distinguishing immediate past from past.

Since Ayer finds all talk of multiplicity of time dimension to be nonsensical (p. 171) – and I am inclined to agree – perhaps he shall be least offended if we say that he would seem to be using, though unwittingly, two different clocks. What is worse, he would seem to use both to explain the one.

General Rules of Language. By *Peter Zinkernagel* (Copenhagen).

As is well known some elementary rules of formal logic have a peculiar power: by breaking them we can reduce any ordinary descriptive statement to nonsense which cannot be used to describe anything. If I say "there is a bird sitting in the tree" and then add "and there is not a bird sitting in the tree" listeners would not understand what I am saying or know what situation I might be trying to describe. Adherence to the law of contradiction seems to be un unavoidable condition for speaking in an intelligible way about what we vaguely and loosely call matters of fact. Of course language is used in many different ways and for many different purposes. We use language for jesting and we use language in poetry and religion. It would certainly be absurd to say that poetry is not poetry because the law of contradiction is not always observed in poetry or that it is forbidden to jest just because we should always adhere to the law of the excluded middle. We may even under certain circumstances use formal contradictions to express difficulties of formulation as when we say "it is and it is not true that the law of contradiction should always be observed". Notwithstanding all such qualifications I still take it that most people would both understand and agree with the proposition that the law of contradiction should be observed in normal discourse.[1] Should anybody doubt this he might be invited to express his views without adhering to the law of contradiction.

In this paper [2] I am going to formulate some rules of language which in my opinion have much the same sort of general validity as have the elementary laws of formal logic: by disobeying them we can reduce any ordinary statement to nonsense which cannot be used to describe matters of fact. I hasten, however, to stress that my attempt is only tentative and that I have by no means succeeded in formulating these rules with the precision characteristic of the elementary rules of formal logic. They have deliberately been given an abstract and general form and they may therefore seem a little strange at first. A few examples will – I hope – suffice to show that we do in fact adhere to these rules when we speak intelligibly together about matters of fact:

1. We must not use names of ordinary things and expressions for possibilities of action independently of each other.

---

[1] See C. A. Campbell "Contradiction: 'Law' or 'Convention'". ("Analysis", March 1958).

[2] For a more detailed discussion reference is made to my book "Omverdensproblemet". Copenhagen 1957.

2. We must not use psychological expressions independently of the personal pronouns.

3. We must not use the personal pronouns independently of designations for bodies.

To illustrate the content of the first rule of language let us see how we describe and how we do not describe such a well known situation as a man sitting before a table on which an inkstand is standing. According to ordinary language we may say for instance that the man is able to move the inkstand and that by doing so he changes his possibilities of action. Having moved the inkstand there are now some things he can do which he could not do before moving it and others which he cannot do but which he could do before he moved it. Having moved the inkstand he can now freely move his hand across the place where the inkstand was formerly standing, but he cannot freely move his hand across the place where it is now standing and so on and so on.

If we want to be in accordance with ordinary language we would not say that he has moved the inkstand but his possibilities of action are left unchanged so that he can move his hand freely across the place to which the inkstand was moved or that he cannot freely move his hand across the place where the inkstand was standing. To say that he has acted (by moving an inkstand) without changing his possibilities of action would almost amount to saying that he has moved the inkstand without moving it and would be in violent disaccord with ordinary language. In like manner it would be strange to talk about a man, who was sitting before a table which did not confine his possibilities of action or indeed to talk about a man who did not confine our own possibilities of action if we tried to move through him. Such a statement would seem to reduce the man to a most ghostly appearance and it is certainly not in accordance with ordinary language to call ghosts ordinary things or people. I hope this is enough to show that we do not in fact use names of ordinary things and expressions for possibilities of action independently of each other in ordinary discourse; that we arrive at most extraordinary statements as soon as we start to use the two sorts of expression independently of each other so that this rule may fairly be said to be a rule characteristic of ordinary language.

The second rule I shall illustrate by another well-known situation, namely that somebody has had a dream and wants to communicate his experience. It would be very much in accordance with ordinary language if he told us "last night I dreamt so and so". It would not be in accordance with ordinary language if he told us "last night there was a dream but neither I nor any one else had that dream" or "last night there was a dream which nobody had". And so with any other psycho-

logical experience which we want to talk about in a normal way. It would be in disaccordance with ordinary language to talk about a great joy which existed but was felt by nobody.

The third rule may be illustrated by the same example as the second rule, namely a man wanting to tell us that he has had a dream. He would be in accordance with ordinary language if he said "last night I was lying on my bed and I dreamt that I was in the Sahara" or even "last night I was lying in my modest room and dreamt that I was a king". He would not be in accordance with ordinary language if he told us that he had had a dream without being either in his bed or in his room or at any other place. He would not be in accordance with ordinary language if he tried to use the word "I" independently of expressions referring to his own body and thereby referring to ordinary things. If he said for instance "I was thinking about some problem but I did not do my thinking either in my room or outside my room for I have not got a body". As names of ordinary things cannot be used independently of expressions for possibilities of action we are back at the first rule of language.

This should be enough to show that we do in fact obey the rules when we use ordinary language in a normal descriptive way. In my opinion they belong among the first rules we learn when we learn to use language and so ought not to be too difficult to recognize. Before giving my reasons for saying that they have the same sort of general validity as have elementary rules of formal logic I should like to investigate our reasons for saying that e.g. the law of contradiction is universally valid. I am not here concerned with the law of contradiction considered as part of a deductive system, but as an unavoidable condition for speaking together in an intelligible manner. Most of us agree that we should try to avoid formal contradictions if we want to put forward unambiguous descriptive or scientific statements. I want to express our reasons for this conviction as clearly as possible. As far as I can see there is only one reason and could only be one reason for saying that observance of the law of contradiction is an indispensable condition for intelligible discourse, namely that we can in fact reduce any descriptive statement to nonsense by violating it. As we cannot use nonsense to describe matters of fact in a straightforward and unambiguous way this one reason is not only strong but conclusive. As far as I can see there could only be one argument, but a conclusive one, against the view that observance of the law of contradiction is an unavoidable condition for speaking together in an unambiguous manner, namely that we could point to descriptive statements which could not be reduced to ambiguity by violating the law of contradiction. Such a test might almost be called a demonstration ad oculos.

Should somebody challenge the view that observance of the law of

contradiction is an indispensable condition for unambiguous communication I do not see that we could argue against him in any other way than by pointing to such an immediate reduction to nonsense of any statement –, which he might care to put forward. If this did not convince him we would be in a rather helpless situation. We could of course point out to him that one consequence of his point of view would be that he might say that his point of view was right and add that it was also wrong (not right) or that it was both true and false (not true). If this, however, did not convince him we would have to acknowledge certain defeat. We could not in any way evaluate his point of view as either right or wrong for the good reason that we could not possibly understand his use of such words as "right" or "wrong". If he said that he could in fact understand a formal contradiction and that a statement expressing this fact by means of a formal contradiction was in fact a true (and false) statement we could not say that this was either true or false because we could not even understand his use of the words "true" and "false", or for that matter "understand".

Now quite a lot of reservations must be made. I have already said that the proposed formulation is only tentative and does not possess the precision of the elementary laws of formal logic. Their validity will be correspondingly less evident. The words with which we are concerned by the logical constants are relatively easily isolated from other words and concepts whereas the words with which we are concerned in the three rules are intimately connected with other words and concepts. Almost any unambiguous statement from the most commonplace utterance to the most subtle theorem in advanced mathematics can be immediately reduced to pure nonsense by disobeying the law of contradiction. Not only is the destructive power of the three rules less apparent but their validity is not in any immediate way presupposed for instance in pure mathematics. Still I shall argue that within a very wide range of statements which I have indicated by the vague word of "descriptive" those rules play a somewhat similar role as do the elementary laws of formal logic.

As in the case of formal logic we only have and only could have one reason for accepting these rules as an indispensable condition for communication, namely that we can in fact reduce statements to nonsense by not adhering to them and that we cannot use language for description if they are not obeyed. It is hardly necessary to give many examples:

"He was scolded by his chief for not having finished his work."

"He was scolded by a chief, who did not (literally) confine his possibilities of action" (first rule). "He was scolded by his chief for not having done something which if he had done it would not have changed anything" (first rule).

"A bad time was had, but it was not had by anybody" (second rule).

"He went through the distressing experience of losing his dog."

"He went through the distressing experience of losing his dog without being anywhere or if he was at some place his body was not there (literally) or if his hody was there it did not confine anybodys possibilities of action" (third and first rule).

Similar transformations could be put to work against any straightforward statement in which occur names of people and of ordinary things or which attempts to describe psychological experiences. It should be stressed that this does not in any way concern our possibilities of having psychological experiences but only the formal conditions for describing them. We may perhaps have any experience whatsoever but we still have to obey certain rules if we want to talk about it in an intelligible way. We may perhaps dream that we do not possess a body but if we want to tell other people about it we would still have to say "last night I lay in my bed and dreamt . . .".

Instead of reducing descriptive statements to nonsense we can investigate our possibilities of description by trying to describe such familiar experiences or situations as speeding along a highway in an automobile or looking at a beautiful sunset. The reader is invited to attempt to describe such situations with the help of expressions like these "we were speeding along a highroad, which did not confine our possibilities of action in an automobile which did not confine our possibilities of action", "we turned the driving-wheel without changing our possibilities of action", "we were looking at a sunset, standing on a mountain, which did not confine our possibilities of action", "we were looking at a sunset without standing on a mountain or being anywhere else", "we were looking at a sunset from a mountain, but our body was not on the mountain", "a beautiful sunset was seen without being seen by anybody".

I believe that the difficulties we would meet in such an attempt are not accidental. They would not diminish but grow with our efforts just as any attempt to dispense with the law of contradiction in descriptions would become more and more hopeless on closer reflection. The indispensability of the first rule becomes particularly apparent if we try to describe the commonplace experience that we can act and thereby change our possibilities of action.[3] The indispensability of all three rules becomes apparent if we investigate our possibilities of describing psychological experiments. In ordinary language we describe

[3] In a paper in "Mind" (October 1955): "On the Problem of Objective Reality as Conceived in the Empiricist Tradition" I have made an attempt to describe such experiences without adhering to the first rule.

a psychological experiment for instance by talking about placing a person before a dark screen with light spots and asking him to tell us what he sees. One may try to describe such an experiment unambiguously without having recourse to the rules by means of expressions like "we asked somebody to tell us what was seen without being seen by anybody", "we placed a person whose body did not confine our possibilities of action on a chair which did not confine our possibilities of action". It should once more be noted that the rules do not in any way concern the contents of psychological experiences but only the formal conditions for describing them: we may devise an experiment which gives people the most extraordinary experiences but we still have to describe the circumstances by obeying the most commonplace rules.

Just as in the case of formal logic, there could only be one argument against the view that the three rules have general validity, namely that we could in fact describe situations or experiences in an unambiguous way without adhering to them. As long as this is impossible we must admit that these rules represent indispensable conditions for unambiguous description. By saying that some rules are indispensable we could hardly mean anything if we did not mean that we could not do without them and I do not see how we could investigate our possibilities of description except by investigating what rules we have to accept if we want to describe anything.

If this really is so. If there could only be one reason for or against the indispensability of certain rules, namely that we could or could not describe anything without respecting them, then it follows that any other reason or argument which might be thought relevant is in fact illusory. As such a view collides violently with what I should call deep-rooted epistemological prejudices I shall consider other arguments at some length.

I shall start with a theory which, if correct, would at once destroy any argument brought forward in this paper, namely the theory of analytical and synthetical propositions. According to it any true proposition is true either because it corresponds to some fact or because it expresses an arbitrary rule of language. Now there can be little doubt that the distinction between "true by correspondence" and "true by convention" is often useful. Our language contains many arbitrary features. If we define "human being" as "rational animal" it is true by definition that human beings are rational animals, whereas a proposition like "there is a bird sitting in the tree" is true, if true, because a bird is in fact sitting in the tree. It is, however, difficult to understand how we could validly infer from this that *all* true propositions belong to one or the other of the two categories. It is difficult to see how we could be entitled to generalize such examples into an all-embracing

theoretical framework, settling in principle all questions about the relationship between language and reality once and for ever. If for no other reason such a conclusion would be dubious because it is easily shown that many of the most difficult problems in modern epistemology stem from acceptance of this theory. It could only be compelling if it had been shown that any epistemologically relevant rule of language was in fact arbitrary and could be dispensed with without impairing the descriptive function of language. If it is a fact that e.g. the law of contradiction is an indispensable condition for unambiguous description we had better accept this fact than try to explain it away to save a theory which is much more dubious. If we admit that we cannot in fact dispense with the law of contradiction, but still call it arbitrary or conventional, we are using the words "conventional" and "arbitrary" in a confusing way. We use such words in a clear way if we refer to circumstances which it is in our power to change. We could use the sound "hest" instead of "horse" and it is therefore correct to say that the use of the sound "horse" is arbitrary or conventional. If, however, we say that we cannot dispense with the law of contradiction but that it is nevertheless arbitrary or conventional we are apt to confuse ourselves. That the law of contradiction is in fact indispensable should be evident as soon as we reflect that any argument against its validity could be reduced to nonsense by violating it. If it is also a fact that we have to obey some other rules if we want to describe anything in an unambiguous manner we had better accept this fact instead of trying to explain it away to save a theory.

It is true that if the theory of analytical and synthetical propositions is correct then the proposed rules of language are arbitrary, just as it is true that if the proposed rules are indispensable then the theory of analytical and synthetical propositions is wrong considered as a universal theory about the relationship between language and reality. How should we decide the issue if not by investigating the facts in the case, namely whether we can or cannot dispense with such rules in descriptions?

A similar line of argument runs somewhat like this: how do we know that what are called indispensable rules of language are not particularly deep-rooted habits of speech, that their apparent necessity derives from their use during countless generations, that they would not or could not have been different if our habits of speech had been different? In support much evidence might be adduced tending to show that certain modes of speech, which by the users were felt to be both natural and necessary, were different among people belonging to a different family of language. My answer would be that it would lead to nonsense to say that e.g. the law of contradiction is only a habit of speech. The law of contradiction is an unavoidable condition for the

use of any expression also the expression "habit of speech". We could only explain what is meant by "habit of speech" by adhering to the law of contradiction. If, having given such an explanation, we add "and the law of contradiction is only a habit of speech" we would be saying something very strange. We would be saying that we could only explain the meaning of "habit of speech" by adhering to a certain law, and adding that this law was an example of that which could not be explained without accepting its validity. Would anybody be able to understand the meaning of such a statement? Could it by any standard of unambiguity be called unambiguous? In like manner I think that we can explain what is meant by "habit of speech" if we adhere to the proposed rules and talk about people and their circumstances in the usual way. But I think we should be led into grave difficulties if we attempted to explain what is meant by "people" without adhering to these rules; let alone explaining what is meant by their habits of speech. If we can only talk about the meaning of "habit of speech" under certain conditions it is not illuminating to call these conditions habits of speech.

Similar difficulties arise if we call logical rules mere laws of thought or say that they are only psychologically valid. If there are certain conditions for any psychological description we cannot without talking nonsense call them psychological.

But might we not consider the proposed rules as definitions which are in fact inherent in ordinary language, so that the reduction to nonsense rests on the law of contradiction? Might we not say that in ordinary language we define a material thing as something which confines our possibilities of action, a psychological expression as something which we can only speak about in connection with the personal pronouns and so on? Having accepted ordinary language we have also accepted these definitions and by rejecting them we both accept and reject them so that we sin not against some new rules but against the old law of formal contradiction?

To this I should say that it is quite true but also quite irrelevant. It is quite true that if we accept certain definitions we cannot without formal contradiction reject them but the question at issue is not this but the nature of the discussed rules or definitions. The question is: are the proposed rules indispensable or are they not? We cannot answer this question by pointing out that we arrive at formal contradiction by both accepting and rejecting them. The rules concern such different words or concepts as "chairs" and "joy" so that it should be immediately clear that they could not possibly be deduced from the elementary laws of formal logic: identity, excluded middle etc. Formal logic gives the rules for using the so-called logical constants and we do not break any rule in formal logic if we decide to define and use

psychological expressions independently of the personal pronouns or if we decide to use the personal pronouns independently of designations for bodies. My point is, however, that we cannot do this without making language unfitted for description just as we cannot disregard the "definitions" of the words "not" and "and" inherent in the law of contradiction without making language unfitted for description.

I hope this is enough to show that if we want to investigate the conditions for unambiguous description or communication we had better do so and not start with any preconceived ideas about the relationship between language and reality or about the nature of logic or about how many logical rules might be found outside those expressed in formal logic. There could only be one reason for or against the indispensability of any proposed rule, namely that it could or could not be dispensed with.

If the three rules must be accepted some important epistemological consequences follow. It follows at once that a proposition like "things do not exist when they are not perceived" is nonsense because it violates the conditions for unambiguous description just as a formal contradiction is nonsense because it violates these conditions. According to the rules the use of names of ordinary things is logically dependent on the use of expressions for possibilities of action, while the use of psychological expressions like "observe" is dependent on personal pronouns and designations for bodies. To say that the existence of material things is dependent on their being observed is to deny these rules by saying that it is the other way round. Many discussions about objective existence start by ascribing a possible meaning to the proposition that things do not exist when not observed and then try to prove that the proposition is either true or false. If the rules are valid, such a procedure amounts to starting a philosophical analysis with nonsense and then hoping that nonsense will change into something which makes sense. We ought to be happy that all hopes in that direction have proved vain. Otherwise we might fear that the opposite could also happen.

Another consequence is that if such rules are valid we could not conceivably explain why they are so just as we cannot explain why we should obey the law of contradiction except by showing that nonsense results if it is not followed. We cannot explain why we should follow the law of contradiction because the sense or meaning of any explanation would itself depend on whether the law is followed or not. In a somewhat similar manner we cannot explain why we have to follow the formulated rules of language. There could, as regards these rules, be no question about the relationship between form and content. If they define conditions for unambiguous description, it would be absurd to say that they deal with language in contrast to reality or with reality in contrast to language.

More on the *Esse* is *Percipi* Principle

by

HASKELL FAIN

(University of Wisconsin, Madison, Wisconsin)

I.

The following assertions claim perhaps the largest share of attention from Berkeley's commentators: "For the existence of an idea consists in being perceived." (*A Treatise Concerning the Principles of Human Knowledge* [=Pr.], Sec. 2). "For as to what is said of the absolute existence of unthinking things, without any relation to their being perceived, that is to me perfectly unintelligible. Their *esse* is *percipi* . . ." (Pr. Sec. 3). "But say you, surely there is nothing easier to imagine trees, for instance, in a park, or books existing in a closet, and no body by to perceive them . . . but what is all this, I beseech you, more than framing in your mind certain ideas which you call *books* and *trees*, and at the same time omitting to frame the idea of any one that may perceive them? But do not you yourself perceive or think of them all the while? This therefore is nothing to the purpose; it only shows that you have the power of imagining or forming ideas in your mind; but it doth not shew that you can conceive it possible, the objects of your thought may exist without the mind; to make out this, it is necessary that you conceive them existing unconceived or unthought of, which is a manifest repugnancy." (Pr. Sec. 23). "It is very obvious, upon the least inquiry into our own thoughts, to know whether it is possible for us to understand what is meant by the *absolute existence of sensible objects in themselves*, or *without the* mind. To me it is evident those words mark out either a direct contradiction or else nothing at all . . . It is on this therefore that I insist, to wit, that the *absolute existence of unthinking* things are words without a meaning, or which include a contradiction." (Pr. Sec. 24.)

What did Berkeley intend by these assertions? What did he suppose to be the relation between them? These are very difficult questions to answer. Berkeley is generally reputed to be one of the clearest of the great philosophers. Indeed on the surface, his ideas seem to be so well formulated and inter-related that his philosophy invites axiomatization, but anyone who tries this procedure (I except Berkeley, who was the first to try; see entry 378 in his *Philosophical Commentaries* ([=P.C.], Luce edition)) will soon observe that the clarity is often deceptive. Berkeley often held many different doctrines simultaneously, apparently without being explicitly aware of this fact (e.g.; See below, p. 72). Naturally, this results in a wide variety of interpretations by the commentators.

Anyone who wishes to analyze the above quotations from Berkeley's *Principles* faces a number of distinct but connected problems: (a) He must give a precise formulation of the *esse* is *percipi* principle; (b) He must explicate carefully the argument (quoted above) in Sec. 23 of Pr.; (c) He must present an exact analysis of the relationship (if any) between the *esse* is *percipi* principle and the Sec. 23 argument; (d) He must make some estimate of what *Berkeley thought* the relationship was between the *esse* is *percipi* principle and the argument in Sec. 23. As an introduction to what follows, I will summarize my own solutions to these problems.

(a) "For the existence of an idea consists in being perceived." This assertion is Berkeley's *esse* is *percipi* principle. As it stands, it is not clear enough for careful analysis. Many commentators (myself included) attempt to reformulate it, using the language of modern logic. In some of these formulations, however, 'exist' appears as a predicate when it ought, in the notation employed, to function as a quantifier. This leads to many difficulties. I reformulate the *esse* is *percipi* principle this way:

$$(y)\ [y \text{ is an idea} \equiv (\exists\ x)\ (x \text{ is a mind} \cdot x \text{ perceives } y)]$$

To avoid unnecessary complications, the expression "x is a mind" will be left out of the above sentence, but is to be implicitly understood; i.e.:

$$(y)\ [y \text{ is an idea} \equiv (\exists x)\ (x \text{ perceives } y)]$$

(b) Some commentators believe that Berkeley's argument in Sec. 23 proves the following to be a contradiction: There exists something which can be truly imagined to be not thought about. I shall show why this is a faulty explication of the Sec. 23 argument. I believe the argument establishes the following: No one can name or describe some object of the senses that no one perceives.

(c) The relationship between the *esse* is *percipi* principle and the Sec. 23 argument I take to be this: No one can *disprove* the *esse* is *percipi* principle (as it applies to objects of the senses) by citing a counter-instance.

(d) Berkeley thought he established in Sec. 23 that the negation of the *esse* is *percipi* principle is either a contradiction or meaningless. Now the same expression cannot be simultaneously meaningless and a contradiction. Furthermore, Berkeley did not show that the negation of the *esse* is *percipi* principle expresses or implies a self-contradiction. More justice can be done to Berkeley's philosophy as a whole by stressing Berkeley's belief that the negation of the *esse* is *percipi* principle is meaningless.

II.

A. N. Prior[1] thinks Berkeley attempted to "establish his idealism" in Sec. 23 and in the first dialogue between Hylas and Philonous. He further thinks that Berkeley in effect erred in the use of an existential quantifier. Prior claims Berkeley established in the Sec. 23 argument that the following is a contradiction:

(1) There exists a y such that x imagines that y is not thought about, and y is not thought about.

According to Prior, Berkeley erroneously thought he showed the following to be a contradiction:

(2) x imagines that there exists a y such that y is not thought

[1] Prior, A. N. "Berkeley in Logical Form", *Theoria* XXI (1955) 117–122. Prior's analysis resembles that of C. Dawes Hicks. See Hicks' *Berkeley*. Benn, London, 1932, pp. 117–18.

about; and there exists a y such that y is not thought about.

Prior constructs a neat proof to show that (1) is a contradiction. However, the proof depends upon interpreting the expression "x is thinking about y" as: For some ϕ, x is imagining that y ϕ's. The expression "y is being thought about" is then interpreted as an existential generalization, with regard to x, of the latter expression; i.e., "y is being thought about" becomes: for some x, for some ϕ, x imagines that y ϕ's.

There are two reasons for not accepting Prior's elegant analysis. First, I feel certain Berkeley would not accept Prior's interpretation of the expression "x is thinking about y" (nor is any such interpretation necessary for explicating Sec. 23). Second, I find no evidence in the text that Berkeley thought (2) a contradiction. If he thought (2) a contradiction, he was presumably trying to prove that the negation of (2) must necessarily be true. Applying suitable transformation rules, the negation of (2) becomes: If x imagines that there exists a y such that y is not thought about, then for all y, y is thought about.

In short, just imagining that something is not thought about entails that everything (including minds) is thought about. So far as I can see, Berkeley never asserted anything like this, nor does it seem to function as an implicit premise in any of his arguments.

To return to the first point – the interpretation of the expression "x is thinking about y". A particular assertion of this form is: Arne is thinking about Randi. According to Prior, this is to be interpreted as: Arne imagines that Randi ϕ's; e.g., Arne imagines that Randi is combing her hair, or is beautiful, etc. "Thinking about" becomes "imagining that". Presumably, then, the expression "to perceive" is to be interpreted "to perceive that". However, Berkeley obviously intended that "x perceives y" be taken as expressing a direct relation between x and y. Perhaps the best evidence of this is how Berkeley uses the expression "x perceives y". He intended the following to be definitionally equivalent:

(3) x perceives y if and only if y is in the mind of x.

(4) x perceives y if and only if x has y.

Syntactically, (3) and (4) make no sense unless "x" and "y" are replaceable by the names of two objects, in which case "perceives" is a genuine dyadic predicate. Hence, Berkeley does not interpret "to perceive" as "to perceive that".

Consider again the expression "Arne is thinking about Randi". Is Berkeley committing a grave philosophical error in supposing that one can think about objects in the same way that one can look at objects? I see no reason why Arne cannot think directly about Randi without thinking any proposition whatever. If this is puzzling, it seems to me that the expression "Arne is looking at Randi" is equally puzzling. Note that one need not suppose that whenever Arne thinks about Randi, he is thinking about some image or picture of Randi that he has, so to speak, in mind's eye. In that case, it would always be false to say such things as "The object I am now thinking about is identical with the object I was looking at a few minutes ago." Quite often, though, we want to say precisely that. In fact, Berkeley's argument in Sec. 23 depends on the possibility that the same object can be both sensed and thought about. This will be shown later. Hume, on the other hand, attempted to draw a sharp distinction between objects looked at (impressions) and objects thought about (ideas). The latter are taken to be "faint resemblances" of the former. Hume's language suggests that Hume thought of ideas as images or pictures of impressions. Thus, according to Hume's doctrine, objects looked at can never be identical with objects thought about, any more than pictures can be identical with the objects they depict. Berkeley never sharply separated impressions and ideas, even though he says in Sec. 1 of the *Principles* that "ideas formed by the help of memory and imagination 'barely represent those originally perceived' . . ." Nothing more is made of the matter.

III.

Marc-Wogau has carefully and judiciously discussed Berkeley's sensationalism, the *esse* is *percipi* principle, and the Sec. 23 ar-

gument.[2] What Marc-Wogau calls "Berkeley's sensationalism" is expressed by Berkeley in such sentences as "That what I see, hear and feel does exist . . . I no more doubt than I do of my own being". (Pr. Sec. 40.) This assertion, and others like it in which Berkeley makes the same claim for ideas of the memory, imagination, and so on, Marc-Wogau calls "the extended sensationalistic thesis". He formulates this thesis as follows:

(5) (x) (x is perceived $\supset$ x exists)

Next, he discusses various ways of formulating the *esse* is *percipi* principle and is in doubt as to which of the following two expressions is best:

(6) (x) (x exists $\supset$ x is perceived)
(7) (x) (x exists $\equiv$ x is perceived)

He decides it is not too important whether Berkeley meant (6) or (7) when he said of unthinking things that their *esse* is *percipi* because if (5) and (6) are true (as Berkeley supposedly thought), then (7) is also true. According to Marc-Wogau, the Sec. 23 argument was intended by Berkeley *to prove* (6) true, and (5) functions as an implicit premise (among others) in this argument. I shall return to this point.

Marc-Wogau considers alternate interpretations of Berkeley's text and explicitly mentions the principles he employs in choosing one interpretation over another. Most of his discussion is clear. Unfortunately, his formulations of Berkeley's theses are not. The trouble is that the word "exist" appears as a predicate when it ought, in the notation he employs, to function as a quantifier. Consider, for example, his formulation of Berkeley's "extended sensationalistic thesis", sentence (5). What would be the negation of (5)? It would read: There exists an x such that x is perceived and x does not exist. This expression in the notation employed, however, is syntactically impossible. Taken literally, Marc-Wogau's formulations do not make sense. However, by examining what he takes to be the negation of (6), one can reformulate (5), (6) and (7) so that they are syntactically

[2] Marc-Wogau, Konrad. "Berkeley's Sensationalism and the *Esse est percipi-Principle*", *Theoria* XXIII (1957) 12–36.

meaningful. The negation of (6) is, according to Marc-Wogau: "There is an idea x, which exists unperceived." i.e., there exists an x, such that x is an idea and x is not perceived. In this case, (6) becomes (6'): For all x, if x is an idea, then x is perceived. Similar changes may be made for (5) and (7). Sentence (5) then becomes (5'): For all x, if x is perceived, then x is an idea.

These reformulations may appear trivial. However, Berkeley's use of the expression "to exist" is important; therefore, care must be taken that the use of this expression in any interpretation of Berkeley is syntactically correct. Otherwise, one compounds confusion. For example, (5') is syntactically correct in Marc-Wogau's notation and (5) is not. It is obvious, however, that (5') does not express what Marc-Wogau calls "Berkeley's extended sensationalistic thesis" . . . "That what I see, hear, and feel does exist . . ." or, as Berkeley puts it in the second *Dialogue*, "That a thing should be really perceived by my senses, and at the same time not really exist, is to me a plain contradiction". Perhaps a better interpretation of the extended sensationalistic thesis is: (y) (y is perceived $\supset$ ($\exists$ x) (x is perceived)). This expression is syntactically well-formed and is moreover a truth of logic, so that its negation is a "plain contradiction".

I do not wish to claim that the latter interpretation, in the notation employed, is the best possible. The point is to show that real confusion can result when syntactically incorrect expressions are introduced. Unlike sentence (6), (5) cannot be patched up without radical change of interpretation or meaning. This is quite unfortunate, for Berkeley's "extended sensationalistic thesis" is supposed by Marc-Wogau to be a missing premise in the Sec. 23 argument; subsequently, Marc-Wogau's interpretation of this argument is difficult to make out.

Marc-Wogau begins his analysis of the Sec. 23 argument by examining four interpretations philosophers have offered. He rejects these for various reasons and then presents his own interpretation, which appears to him to be "the most probable" of those discussed. His interpretation consists in presenting three premises that can function as suppressed premises in Berkeley's argument in Sec. 23, the argument Marc-Wogau claims is offered

as a proof of the *esse* is *percipi* principle, (6). The three premises are:

(a) "A proposition p is proved, if the proposition *I assume that p is false, is a contradiction.*"
(b) "To suppose something about x implies to perceive x."
(c) "I suppose that x is unperceived implies that x is unperceived."

Marc-Wogau admits there is no evidence in Berkeley's writings to show that Berkeley held (a). Also, strangely enough, Marc-Wogau claims at this point that (c) is Berkeley's extended sensationalistic thesis. Certainly, (c) does not resemble (5) or (5'). Ignoring this, however, I fail to see how these premises enable one to prove that all ideas are perceived. According to (a), the way to prove that all ideas are perceived is to show that the sentence "I assume (suppose) that some ideas are unperceived" is a contradiction. By (b), if I suppose that an idea is unperceived, then I perceive an idea. By (c), if I suppose that an idea is unperceived, then an idea is unperceived. Surely, however, there is no contradiction here. One cannot show, from the premises as stated, that the same idea is both perceived and unperceived. The fault lies in the ambiguity of the premises. Consider Marc-Wogau's own précis of the above: ". . . the proposition *I suppose that some idea x, exists unperceived,* is contradictory, because it implies that x is both perceived and not perceived." [8] In its first occurrence, "x" is functioning as a bound variable, but in the second occurrence "x" seems to be used as a name of an object.

IV.

In the *Principles*, Berkeley "translates" expressions containing the word "exist" in various ways. Consider first the expression "There exists a table". The following kinds of translations are offered by Berkeley: (a) I perceive a table. (b) Under certain conditions I would perceive a table. (c) Someone perceives a

[8] Marc-Wogau, Konrad, *Loc. Cit.*, p. 34.

table. (d) Someone under certain conditions would perceive a table. (e) God perceives a table.

Not all these translations are compatible, and Berkeley's jumps from one to the other make it difficult to select the most important one. As Marc-Wogau indicates, if (b) or (d) were the most important, Berkeley should have said of ideas that their *esse* is *posse percipi*. I think Marc-Wogau correct in saying that Berkeley abandoned (b) and (d). After all, Berkeley believed Someone (God) was always around to perceive unthinking things. I choose (c) as Berkeley's most important translation of the sentence "There exists a table". (c) is compatible with (a) and (e), being nothing more then an existential generalization of (a) and (e). I assume that Berkeley had such translations as (c) in mind when he said of unthinking things or ideas that their *esse* is *percipi*.

Consider next how Berkeley dealt with such expressions as "I exist", "God exists". He did not want to say of thinking things (minds) that their *esse* is *percipi*, for the simple reason that he believed minds or spirits were not perceived but nonetheless existed. In his *Philosophical Commentaries*, he said "Existence is *percipi* or *percipere*". (P. C. Entry 429) He apparently agreed with Descartes that "I am, I exist . . . just when I think, for it might possibly be the case if I ceased entirely to think, that I should likewise cease altogether to exist". (*Meditation* II) However, where Descartes said "I think", Berkeley would have substituted "I perceive". The *esse* of minds, then, is to perceive. (Cf. Sec. 98, Pr.)

That Berkeley drew a sharp distinction between such sentences as "Tables exist" and "Minds exist" is very important in deciding whether Berkeley attempted to *demonstrate* in the *Principles* that the *esse* of ideas is *percipi*. To provide such a demonstration, he would have had to be prepared to say that the existence of all unperceived things (including minds) is a contradiction, to say that the expression "there exists" is synonymous with the expression "there is perceived". I believe Berkeley was aware of this – that he once seriously considered demonstrating the *esse* is *percipi* principle but (more or less) gave up the idea, probably

because he couldn't as a religious person countenance the non-existence of souls. Briefly, I believe Berkeley once seriously considered a position similar to the one Hume later espoused.

The evidence to which I appeal in support of my interpretation is a remarkable set of entries in *Philosophical Commentaries*, 577–581 and 637. The main points in these entries are that the words "mind" or "soul" either designate "congeries of Perceptions" (P. C. entry 580) or else have no meaning, and that the existence of unperceived things is a contradiction.[4]

In entry 586, Berkeley states "I shall Demonstrate all my Doctrines". In entry 589, he says "There was a smell i.e. there was a smell perceived. Thus we see that common speech confirms my doctrine".

How are these entries related to one another? I believe that at the time he made these entries, Berkeley was considering the notion that the *esse* of everything was *percipi*. The following formulation, using the symbols of quantification, expresses this notion:

(8) (x) (x is perceived).

The demonstration of (8) which I think Berkeley had in mind was a *reductio ad absurdum*. He intended to demonstrate that the negation of (8) was a contradiction. The negation of (8) is: (∃ x) (x is not perceived). Why did he think the negation of (8) a contradiction? Entry 589 (quoted above) provides a clue. I think Berkeley proposed in that entry, on the presumed authority of common speech, that the expression "there exists" is synonymous with the expression "there is perceived". In this case, the negation of (8) by replacement becomes: There is perceived an x such that x is not perceived. If this is a contradiction, (8) is demonstrated.[5]

[4] "Consult, ransack yr Understanding wt find you there besides several perceptions or thoughts. Wt mean you by the word mind you must mean something that you perceive or yt you do not perceive. a thing not perceived is a contradiction. to mean (also) a thing you do not perceive is a contradiction. We are in all this matter strangely abused by words." (P.C. Entry 579.)

[5] Assuming that the sentence "There is perceived an x such that x is

In the *Principles* and *Dialogues*, Berkeley could not allow "there exists . . ." to be replaced by "there is perceived . . .", for in the sentence "there exists a mind", the replacement would yield what Berkeley held to be a false sentence – minds are not supposed to be perceived. However, when considering a formal demonstration of the *esse* is *percipi* principle, he held there was no reason to suppose ". . . there must be a thinking substance. Something unknown which perceives & ties together the Ideas." (P. C. entry 637) His change of position necessitated a change of tactics. Instead of trying a formal demonstration of the *esse* is *percipi* principle (now restricted only to ideas), he asked those who thought his doctrine false to prove their case by producing a counter-instance.

What, then, is the *esse* is *percipi* principle as it occurs in the *Dialogues* and *Principles*? What does Berkeley mean when he states in Pr. Sec. 3, "For as to what is said of the absolute existence of unthinking things, without any relation to their being perceived, that is to me perfectly unintelligible. Their *esse* is *percipi* . . ."?

The principle may be formulated in various ways. I choose a formulation that allows for a relatively simple explication of the important passage which occurs in Pr. Sec. 23. The *esse* is *percipi* principle may be stated as:

(9) (y) [y is an idea $\equiv$ ($\exists$ x) (x perceives y)].

Berkeley assumed part of the principle would be accepted with-

not perceived" may be interpreted as: something is perceived and not perceived, then it is certainly a contradiction. However the negation of this latter sentence is not sentence (8) but the logically true sentence: (x) (x is perceived $\supset$ x is perceived). The problem here involves the syntax of the expression "there exists". As is well known, the universal quantifier can be defined by means of the existential quantifier and the expression 'not'. If furthermore, the expression 'there exists' is synonymous with the expression 'there is perceived', then sentence (8) means: It is not the case that there is perceived an x such that x is not perceived. However, this is hardly what one would normally mean in saying that everything is perceived! In short, even if we were to grant that 'there exists' is synonymous with 'there is perceived' (and there are good reasons for not granting this), sentence (8) becomes trivially true.

out special pleading. For example, part of the above equivalence reads:

$$(y)\ [(\exists x)\ (x \text{ perceives } y) \supset y \text{ is an idea})]$$

This is logically equivalent to:

$$(y)\ (x)\ (x \text{ perceives } y \supset y \text{ is an idea}),$$

which may be stated as: No one ever perceives anything that is not an idea; or, more loosely, what is perceived is always an idea. In this view, Berkeley was in fundamental agreement with Locke in saying that we never directly perceive material objects. The arguments advanced in support of the view are sufficiently well-known and for present purposes require no special comment. It is the other half of the equivalence that Berkeley felt needed special treatment. Even here, however, he thought he would be easily granted certain things. "That neither our thoughts, nor passions, nor ideas formed by the imagination, exist without the mind, is what every body will allow." (Pr. Sec. 3.) In the notation used here, the following is what "every body will allow":

(10) $(y)\ [(y \text{ is a thought } v\ y \text{ is a passion}) \vee y \text{ is an imagining}) \supset (\exists x)\ (y \text{ is in the mind of } x)]$. By (3), sentence (10) becomes

(11) $(y)\ [(y \text{ is a thought } v\ y \text{ is a passion}) \vee y \text{ is an imagining}) \supset (\exists x)\ (x \text{ perceives } y)]$.

Berkeley never stated explicitly why he thought everyone would allow him (10). No doubt the notion of mind was so tied up with that of idea that it seemed no more possible to have ideas without minds which "contained" them than colors which were not extended. (11) was taken by Berkeley to be synonymous with (10). However, (11) is not as "obvious" as (10). In fact, the Locke-Leibniz controversy over the existence of innate ideas can be interpreted as the question of whether (11) is true. Both Leibniz and Locke would have accepted (10). Locke, when not jousting with such windmills as the question "Do babies know the multiplication table?", held the crux of the problem of innate ideas to be: Can we have ideas in our minds of which

we are not aware, which we do not perceive. He answered, of course, in the negative. Leibniz, on the other hand, argued that it was perfectly possible to have an idea and not be aware of it. In illustration, he maintained we have memories of which we are not aware, or do not perceive, at all times. In short, Locke – as well as Berkeley and Hume – held (10) to be synonymous with (11). Leibniz did not. He thought (10) true and (11) false. Interestingly enough, Berkeley never seriously considered memory ideas as a separate problem. He rarely mentioned them. (For that matter, neither Locke nor Hume paid enough attention to the problems of memory phenomena.)

Let us grant, as Berkeley wished to do, that (10) and (11) are synonymous and true. In the *Dialogues,* he in effect added "pleasure" and "pain" to the set of disjuncts making up the antecedent clause of (11). Let us also grant as true the amended version. We still have not yet accepted the *esse* is *percipi* principle. As he used the term, "idea" applied not only to thoughts, passions, imaginations, pleasures, pains and memories, but also to "objects of the senses". In order, then, to get us to accept the *esse* is *percipi* principle, Berkeley must show that the principle holds when "idea" refers to objects of the senses. This is what he tried to do in Sec. 23 Pr. There he maintained that the following is true:

(12) (y) [y is an object of the senses $\supset$ ($\exists$ x) (x perceives y)].

As I stated earlier, I do not think Berkeley really tried to demonstrate (12). It is a mistake, therefore, to search for suppressed premises or logical errors. All he asked was that the reader try to prove (12) false by producing a counter-instance. Or, to put it another way: If we have reason for believing (12) false, we must have reason for believing its negation true. The negation of (12) is:

(13) ($\exists$ y) [y is an object of the senses $\cdot$ (x) (x does not perceive y)]

i.e., there exists an object of the senses that no one perceives. Berkeley asked our reason for believing (13) true. He showed us

that we cannot prove (13) true by naming or describing some sense object no one perceives. Suppose, for example, that I put a book into an empty closet, close the door, and assert "The book in my closet is an object of the senses, and no one perceives the book in my closet." If my assertion is true, I will have proved (13) true because my assertion logically implies (13). Is, however, my assertion true? Berkeley maintained not, because I perceive the book in my closet at the time I make my assertion. Hence, it is false that no one perceives the book in my closet, and my whole assertion is then false, having one false conjunct. If I retort that I am not, at the time, "sensing" the book in my closet, his reply is that I am still thinking about the book and thinking, or conceiving, "is a species of perceiving".

It is easy to misunderstand Berkeley, by thinking that his argument plays on an ambiguity of the expression "to perceive" and is therefore mere sophistry. An attempted argument against Berkeley's point is as follows: Just as the word "idea" ambiguously designates either thoughts, passions, imaginings, memories, pleasures, pains or sense-data, so the expression "to perceive" ambiguously designates thinking, imagining, sensing, and so on. Berkeley's *esse* is *percipi* principle, so this argument runs, is ambiguous as it stands. It contains two ambiguous terms, "idea" and "perceive". One might think that to avoid this ambiguity, it is necessary to restate the *esse* is *percipi* principle for each type of object and each correlative mental activity:

(14) (y) [y is a pain≡(∃ x) (x feels y)].
(15) (y) [y is a thought≡(∃ x (x thinks y)]

and so on. In particular, the following:

(16) (y) [y is an object of the senses≡(∃ x) (x senses y)].

Berkeley's argument in Sec. 23, it might be claimed, consists merely in shifting the meaning of "perceive" from the sense it has in (16) to the sense it has in (15). When, for example, I claim I am not now perceiving (i.e., sensing) the book in my closet, Berkeley's retort is that I am still perceiving (i.e., thinking about) the book in my closet. However, it is (16) and not (15) that is supposedly being challenged in Sec. 23, and Berkeley

tricks us into thinking he has met the challenge when actually all he has shown is that we cannot prove (15) false.

The above argument would be cogent if Berkeley had ever said it was impossible to sense and think about the same object. So far as I can tell, he never said this, and I see no reason why he should. When I say "I see the book in my closet at time t" and when I say "I am thinking about the book in my closet at t", there is no reason why the expression "the book in my closet at time t" can not designate the same object in both sentences. The *esse* is *percipi* principle need not be restated for each type of object corresponding to each type of mental activity; the same object can be sensed and thought about.

What, then, did Berkeley show in Sec. 23? He showed, it seems to me, that one cannot give a "constructive" proof of (13). Sentence (13) is of the form "$(\exists x)(\phi x)$" and Berkeley demanded that we prove (13) by means of a true sentence of the form "ϕa", where "a" is an individual constant. We cannot do this, for when we name or describe some object we claim to be unperceived, we are presumably thinking about that object.

In all this (as in his mathematical essays), Berkeley is very much in the spirit of those contemporary mathematicians and philosophers who refuse to allow the existence of a mathematical object having a certain property until one can 'produce' the required object. Consider, for example, the objection some have to accepting the axiom of choice. The axiom of choice may be stated as: if T is a collection of disjoint, non-empty sets Se, then *there exists* a set R which contains one and only one member Xe of each Se. Now some people will accept the axiom of choice if and only if it is possible, for any T, to construct R. *e.g.*, If T is a collection of disjoint pairs of unequal integers, then one can construct a set R by taking the smaller integer in each pair. But this is tantamount to describing or naming a set R, the members of which consist of one and only one integer from each pair. Thus, the proof that there exists a set R, the members of which consist of exactly one integer from each pair, amounts to naming or describing such a set. Suppose, however, T is an infinite collection of non-empty disjoint pairs whose members are indistinguish-

able. Does there in this case exist a set R? There does according to the axiom of choice, but no such set can be named or described because no rule can be given for selecting one member of each pair – all elements are indistinguishable from one another.

Some who reject the axiom of choice do so precisely because in cases like this no construction proof can be given that there exists an R, the members of which consist of exactly one element from each pair. No set R, in this case, can be named or described. Berkeley's Sec. 23 argument proceeds along similar lines. To show that there exists an object having a certain property, one must be able to name or describe such an object. If we wish to show that there exists an object of the senses that no one perceives, we must name or describe such an object. Any object that *we* name or describe, however, *we* perceive. Hence, we cannot name or describe an object that no one perceives. We cannot then prove (by "construction") the negation of the *esse* is *percipi* principle (sentence 13). To put it another way, no one can produce a counter-instance to the *esse* is *percipi* principle.

What follows from all this? Berkeley would have been mistaken if he thought that to show a sentence cannot be proved is to prove its negation true, that to show (13) cannot be proved is the same as proving (12). The question, however, is what Berkeley thought he showed in Sec. 23. This is not an easy question to answer. Most commentators believe Berkeley thought he was proving the *esse* is *percipi* principle as it applies to "objects of the senses". This opinion is in part based on a passage that occurs in Sec. 24 Pr.: "To me it is evident those words [*the absolute existence of sensible objects in themselves,* or *without the mind*] mark out either a direct contradiction or else . . . are words without a meaning."

My opinion is that Berkeley himself was not clear about what he established in Sec. 23. I have suggested that Berkeley once thought he could demonstrate the *esse* is *percipi* principle but later abandoned the idea. However, the passage in Sec. 24 would seem to indicate that he didn't completely abandon the idea after all. This ambiguity on Berkeley's part is reflected in his *Philosophical Commentaries,* wherein he sometimes states he will

demonstrate his doctrines and sometimes says "I must not pretend to promise much of Demonstration, I must cancell all passages that look like that sort of Pride, that raising of Expectation in my Readers." (P.C. 858.)

It is evident that sentence (13) cannot be contradictory and meaningless at the same time. Furthermore, it is evident that (13) is not a contradiction. I think it does more justice to Berkeley's philosophy as a whole to stress the expression "or else ... are words without a meaning" in the passage quoted from Sec. 24. According to this interpretation, what Berkeley is saying is that no sentence of the form "$(\exists x)\ (\phi x)$" is meaningful unless one can prove such a sentence by means of a true sentence of the form "ϕa", where "a" names or describes a particular object. For example, the sentence "there exists a cat" is meaningful (according to this interpretation) if some particular cat can be named or described. This interpretation is in harmony with Berkeley's nominalism and his attack on the doctrine of abstract ideas. According to this analysis, what Berkeley thought he showed in Sec. 23 was that the sentence "There exists an object of the senses that no one perceives" is meaningless, because no particular object having the requisite character can be produced. There are difficulties with this view but these difficulties are philosophical, not logical, and they involve to a certain extent the problem of nominalism itself.

$\aleph_0$-categoricity in first-order predicate calculus[1]

by

LARS SVENONIUS
(University of Uppsala)

I. *Basic concepts*

1. *Deductive systems.* By a *deductive system,* or simply a *system,* we will mean a pair (L, Σ) of a language L and a class Σ of sentences in L, closed under the consequence relation. The language L will always be assumed to be a language in first-order predicate calculus with identity, without predicate variables and without individual constants, and with a finite or denumerable number of predicate constants. The system (L, Σ) will in most cases be assumed to be complete and consistent, i.e. for any sentence P of L either P or $\sim P$ belongs to Σ but not both.

2. *Structures, models.* A *structure* associated with a language L may be defined in various different ways. It is enough here to state those properties of a structure that we will use.

Characterization. An L-structure M determines a class of objects U_M called the *universe of* M, and assigns to every predicate constant R in L of degree n a class of n-sequences from U_M, called the *extension* of R in M.

Def. When L is a language containing an identity relation I, we call M a *reduced structure,* if no pair (a, b), where a and b are different objects of U_M, belongs to the extension of I in M.

[1] After this paper was completed, I was informed that an essential part of my results was already known. As appears from Mostowski's paper [3], T 10 has been proved by Ryll-Nardzewski, although his proof has not yet been published. From Mostowski's paper it is apparent that Ryll-Nardzewski's proof of the implication from (b) to (a) is the same as in this paper. Concerning the more difficult proof of the converse implication, Mostowski gives no information.

If M is any L-structure, there are obvious meanings of the phrase "P is true in M" for any sentence P in L, and of the phrase "$(a_1, \ldots, a_n)$ satisfies $A(x_{i1}, \ldots, x_{in})$", where $(a_1, \ldots, a_n)$ is a sequence of elements of U_M and $A(x_{i1}, \ldots, x_{in})$ is some formula of L containing no free variables other than $x_{i1}, \ldots, x_{in}$, and where a certain correlation between the variables and the objects is presupposed. A *model* of a system (L, Σ) is an L-structure M for which all the sentences of Σ are true. If M is any L-structure, there is a uniquely determined system (L, Σ_M) which is complete and such that M is a model of (L, Σ_M).

3. *Isomorphism, $\aleph_0$-categoricity.*

Def. Let M_1 and M_2 be two L-structures. By a *partial isomorphism relation* between M_1 and M_2 we understand a two-place relation Φ with its domain D_1 included in U_{M1}, and the converse domain D_2 included in U_{M2}, and such that whenever A is a formula in L and $\Phi a_i b_i$ for $i=1, \ldots, n$, then $(a_1, \ldots, a_n)$ satisfies A in M_1 if and only if $(b_1, \ldots, b_n)$ satisfies A in M_2. If a partial isomorphism relation Φ between M_1 and M_2 has U_{M1} for domain and U_{M2} for converse domain, Φ is called a *total* isomorphism and M_1 and M_2 are said to be *isomorphic*. If a partial isomorphism Φ between M_1 and M_2 has U_{M1} for its domain, M_2 is said to be an *arithmetical extension* of M_1 [5]. A system (L, Σ) is called *categorical* if all its models are isomorphic, and *$\aleph_0$-categorical*, if all its denumerable models are isomorphic.

Note. If the structures M_1 and M_2 are reduced structures, any partial isomorphism relation must be one-one. Otherwise it need not be. It is easy to see that to any given structure M there corresponds some structure M' which is reduced and which is isomorphic to M.

4. *Condition-sets.* A set Γ of formulas in L is called a *condition-set in* $(x_{i1}, x_{i2}, \ldots, x_{in})$, and a *condition-set of degree n*, if it contains no variables free other than $x_{i1}, \ldots, x_{in}$. (When we speak of *condition-sets* simply, we always mean condition-sets of some finite degree.) When speaking about formulas containing free variables we need to introduce a special consequence relation.

Def. If Γ_1 and Γ_2 are classes of formulas in L, we say that Γ_2

is a *c-consequence* [2] of Γ_1 if for every formula A of Γ_2 there are formulas $B_1, \ldots, B_n$ in Γ_1 such that the formula

$$(B_1 \cdot B_2 \ldots B_n) \supset A$$

is a valid formula in predicate calculus.

In other words: A c-consequence of Γ_1 is a class of sentences which one can derive from Γ_1 treating the free variables of Γ_1 as constants. When Γ_1 does not contain any free variables, the c-consequences of Γ_1 are the same as the consequences in the usual meaning. In the following, when using the term *consequence*, I will always mean c-consequence.

A condition-set Γ is called *c-consistent in* (L, Σ), or briefly *consistent in* (L, Σ), if no contradiction is a c-consequence of $\Sigma \cup \Gamma$; Γ is called *maximal c-consistent* (or *maximal consistent*), if it is c-consistent, contains the free variables $x_1, \ldots, x_n$, and is included in no other consistent condition-set in $x_1 \ldots, x_n$. (Clearly, any maximal consistent condition-set is included in some consistent condition-set of higher degree.)

If $a_1, \ldots, a_n$ are individual constants not belonging to L we can speak of *condition-sets in the symbols* $a_1, \ldots, a_n$ in the same way as if these symbols were variables.

II. *Condition-sets with finite basis*

When (L, Σ) is a complete consistent system, there are close connections between the maximal consistent condition-sets in (L, Σ), and the models of the system. We state first some obvious results:

T 1. a) Let (L, Σ) be a complete consistent system, M a model of (L, Σ) and $(a_1, \ldots, a_n)$ a sequence of elements in M. Then the set of those conditions in the variables $x_1, \ldots, x_n$, which are satisfied by $(a_1, \ldots, a_n)$, is a maximal consistent condition-set in (L, Σ).

b) If (L, Σ) is a complete consistent system, and Γ is a maximal consistent condition-set in (L, Σ), then there is some

[2] A similar relation is called I-consequence in Los [2].

denumerable model M of (L, Σ) in which Γ is satisfied by some sequence.

On the other hand, as we shall see (T 7 and T 8), a model of a complete consistent system (L, Σ) does not in general exemplify all the maximal consistent condition-sets of (L, Σ). Condition-sets of a special type, those "with a finite basis", must however always be satisfied. We will now define this concept.

Def. Let (L, Σ) be a system, and Γ a condition-set in (L, Σ). A subclass Γ' of Γ is called a *basis of* Γ *in* (L, Σ), if Σ, $\Gamma' \rightarrow \Gamma$. (This symbolism shall be interpreted to state that Γ is a c-consequence of $\Sigma \cup \Gamma'$.) If Γ' is a finite subclass, it is called a *finite basis*.

If Γ is maximal consistent and has a finite basis, then it evidently even has a one-element basis. About condition-sets with finite basis we have the following result already announced:

T 2. If (L, Σ) is a complete consistent system, and Γ is a consistent condition-set with a finite basis in (L, Σ), then Γ must be satisfied in every model of (L, Σ).

As we saw in *T 1*, every sequence $(a_1, \ldots, a_n)$ of elements in a structure M determines a maximal consistent condition-set. It follows that we may apply the concept of basis to such sequences.

Def. Let M be any structure, (L, Σ) the corresponding system, $(a_1, \ldots, a_n)$ some sequence of elements in M, and Γ the maximal consistent condition-set in $(x_1, \ldots, x_n)$ determined by $(a_1, \ldots, a_n)$. Any basis of the set Γ in (L, Σ) will also be called a *basis of the sequence* $(a_1, \ldots, a_n)$ *in* M. If M is a structure in which every finite element-sequence has a finite basis, we say that M has *finite character*.

T 3. Let (L, Σ) be a complete consistent system, and M_1 and M_2 two denumerable models of (L, Σ), both with finite character. Then M_1 and M_2 are isomorphic.

Proof. To simplify the proof, we will assume that M_1 and M_2 are reduced structures. Since every structure M is isomorphic to some reduced structure M', and the reduced structure M' has finite character if and only if M has, this assumption does not make the proof less general.

a) We first observe that we can find some partial isomorphism Φ between M_1 and M_2 with any given finite subclass F of U_{M1} as its domain. Let $(a_1, \ldots, a_n)$ be an enumeration of the elements of F, and let Γ be the corresponding maximal consistent condition-set. Since M_1 has finite character, Γ has a finite basis and is therefore satisfied by some sequence $(b_1, \ldots, b_n)$ in M_2 (*T* 2). The correlation Φ: $a_i \longleftrightarrow b_i$ is evidently a partial isomorphism.

b) We can also show that, under the stated conditions, any finite partial isomorphism between M_1 and M_2 can be extended to include any additional element of U_{M1} or U_{M2}. Let $a_1, \ldots, a_n$ be the elements in the domain of Φ, and $b_1, \ldots, b_n$ the corresponding elements in the converse domain. Let a be some additional element of U_{M1}. Since M_1 has finite character, there is some $(x_1, \ldots, x_{n+1})$-condition A in L which is a basis of the sequence $(a_1, \ldots, a_n\ a)$ in M_1. Then the condition $(Ey)A(x_1, \ldots, x_n\ y)$ is satisfied by $(a_1, \ldots, a_n)$ in M_1, and then also by $(b_1, \ldots, b_n)$ in M_2, since Φ is partial isomorphism. This means that there is an element b in U_{M2}, such that $(b_1, \ldots, b_n, b)$ satisfies the condition A. Since the set deducible from A is maximal consistent, the sequences $(a_1, \ldots, a_n, a)$ and $(b_1, \ldots, b_n, b)$ satisfy exactly the same conditions, and we see that the correlation extended with the correlation $a \longleftrightarrow b$ is again a partial isomorphism.

c) We can now prove the assertion of the theorem. By a) and b) we can find a sequence $\Phi_1, \ldots, \Phi_m \ldots$ of finite partial isomorphisms, where every Φ_{m+1} is an extension of the preceding Φ_m. Since M_1 and M_2 were assumed to be denumerable, we can evidently construct the sequence so that every element of U_{M1} or U_{M2} is included in Φ_n for some n. The sum of all these correlations must now be an isomorphism between M_1 and M_2. The domain is U_{M1} and the converse domain is U_{M2}; and since any finite sequences s_1 from U_{M1} and s_2 from U_{M2}, which correspond to each other by Φ, are already correlated by one of the partial isomorphism Φ_m, they must satisfy exactly the same conditions in L.

By the same method of proof we can also prove the following: *T* 4. If (L, Σ) is a complete consistent system, and M_0 is a

denumerable model of (L, Σ) with finite character, then every other model M of (L, Σ) is an arithmetical extension of M_0.

It is clear, that if M is not itself denumerable and with finite character, this relation cannot hold the other way. So we get the following consequence of T 3–4:

T 5. If (L, Σ) is a complete consistent system, and has a model M_0 of finite character, then this model M_0 is within isomorphism uniquely characterized as the model of which every other model is an arithmetical extension.

The following theorem states a sufficient condition for $\aleph_0$-categoricity. We show later (T 9) that this condition is also necessary.

T 6. Let (L, Σ) be a complete consistent system. If every maximal consistent condition-set in (L, Σ) has a finite basis, then (L, Σ) is $\aleph_0$-categorical.

The theorem is an immediate consequence of T 4 by the observation (T 1b) that every model of (L, Σ) has finite character.

III. *Condition-sets without finite basis*

Our main goal is now to prove the converse of T 6. In other words, we want to show that if a complete consistent system (L, Σ) contains some maximal consistent condition-set without finite basis, then it is not $\aleph_0$-categorical. We use different methods of proof, according as the set of different maximal consistent condition-sets is denumerable or not.

T 7. Let (L, Σ) be a complete consistent system, such that the number of different maximal consistent condition-sets is nondenumerable. Then (L, Σ) is not $\aleph_0$-categorical.

Note. There can be only a denumerable number of condition-sets with a finite basis. So if there are nondenumerably many maximal consistent condition-sets, there are also nondenumerably many maximal consistent condition-sets without finite basis.

Proof of T 7. In a denumerable model, there is only a denumerable number of finite sequences. Therefore for any denumerable model M_1 of (L, Σ) there is some maximal consistent condition-set Γ which is not satisfied in M_1. But according to T 1*b*

there is some denumerable model M_2 of (L, Σ) in which Γ is satisfied. M_1 and M_2 are not isomorphic, and so (L, Σ) is not $\aleph_0$-categorical.

The proof in the denumerable case is more complicated, but it also gives us a stronger result.

T 8. Let (L, Σ) be a complete consistent system such that the set $\mathfrak{G}$ of the maximal consistent condition-sets without finite basis in (L, Σ) is (at most) denumerable. Then there is a denumerable model of (L, Σ) with finite character, i.e. a model in which none of the condition-sets $\Gamma \varepsilon \mathfrak{G}$ is satisfied.

For reference in the following proof we will first list two simple lemmas.

Lemma 1. If $\Phi, A(a) \rightarrow \Psi$ where $A(a)$ is a sentence containing the individual constant a and Φ and Ψ are sets of sentences where a does not occur, then $\Phi, (Ex)A(x) \rightarrow \Psi$.

Lemma 2. If in a structure M a sequence $(a_1, \ldots, a_n)$ is without finite basis, then there is some subsequence $(a_{i1}, \ldots, a_{ik})$ without repetitions, which is also without finite basis.

Example. Let a and b be different elements in the structure M. The sequence $(a\ a\ b\ b\ b)$ is a sequence in M with repetitions and $(a\ b)$ a subsequence without repetitions. We assume that $(a\ b)$ has a finite basis. This means that there is some condition $A(x_1\ x_2)$ which is satisfied by $(a\ b)$ and from which every other condition $B(x_1\ x_2)$ satisfied by $(a\ b)$ is a consequence. If we define the condition $C(x_1\ x_2\ x_3\ x_4\ x_5)$ as $A(x_1\ x_3)$. $x_1 = x_2 \cdot x_3 = x_4 = x_5$, this is easily seen to be a basis for the sequence $(a\ a\ b\ b\ b)$.

Proof of T 8. In the construction of the model we will follow the procedure in Henkin's completeness proof [1], with some additional qualifications.

Henkin's construction of a model of a complete consistent system (L_0, Σ_0) can be described as follows: We suppose that we have an infinite number of infinite sequences $s_1\ s_2 \ldots$ of individual constants. We may write $s_1 = (a_{11}\ a_{12}\ a_{13} \ldots)$, $s_2 = (a_{21}\ a_{22}\ a_{23} \ldots)$, etc. The total set of individual constants a_{ij} we may call S. We form a series of successive extensions (L_1, Σ_1), (L_2, Σ_2) $\ldots$ of (L_0, Σ_0). The construction of each system (L_k, Σ_k) from

the preceding system (L_{k-1}, Σ_{k-1}) can be described as the following three-step procedure:

k *a*) L_k is formed from L_{k-1} by adjunction of the symbols $a_{k1}, a_{k2}, \ldots$ of s_k to L_{k-1}.

k *b*) (L_k, Σ'_{k-1}) is a consistent system formed from (L_k, Σ_{k-1}) by adding to Σ_{k-1} a sentence $A(a_{ki})$ corresponding to every sentence $(Ex)A(x)$ in Σ_{k-1}.

k*c*) (L_k, Σ_k) is formed by extending (L, Σ'_{k-1}) to a complete consistent system.

The sum Σ_ω of all the sets Σ_n determines a model M with its elements in a one-one denotation correlation with the symbols of S and such that all the sentences of Σ_ω are true in M.

We observe: If $\alpha_1, \ldots, \alpha_n$ are individual constants adjoined to L_k (i.e. belonging to some of $s_1, \ldots, s_k$) then Σ_k contains a maximal consistent condition-set in the symbols $(\alpha_1, \ldots, \alpha_n)$, which is satisfied by the corresponding objects. We may denote this condition-set $\Sigma(\alpha_1, \ldots, \alpha_n)$. If M has finite character, then evidently every such set $\Sigma(\alpha_1, \ldots, \alpha_n)$ has a finite basis in (L, Σ).

We will now show that by performing the completion processes 1*c*, 2*c*, etc. in a special way we will obtain a model M with finite character.

If Γ is a condition-set $\{A_i(x_1, \ldots, x_n)\}$, and $(\alpha_1, \ldots, \alpha_n)$ is a sequence of symbols in S, then the corresponding elements in M will satisfy Γ if and only if all $A_i(\alpha_1, \ldots, \alpha_n)$ belong to Σ_ω. A sentence $\sim A_i(\alpha_1, \ldots, \alpha_n)$ will be called a *negative assignment to the pair* $\{\Gamma, (\alpha_1, \ldots, \alpha_n)\}$. In order that the model M has finite character, it is necessary and sufficient that no element-sequence in M shall satisfy any condition-set $\Gamma \varepsilon \mathfrak{G}$. But in view of Lemma 2 we may restrict ourselves to repetition-free sequences. We get then: M has finite character, if and only if Σ_ω contains some negative assignment to every pair (Γ, t), where $\Gamma \varepsilon \mathfrak{G}$ and t is a repetition-free symbol-sequence from S of appropriate degree. (The restriction to repetition-free symbol-sequences is made to avoid some trivial complications in the proof.)

If by S_k we denote the subset of S that is adjoined to L_k, namely the set of the elements from $s_1, \ldots, s_k$, we can give the

following equivalent formulation: The model M has finite character if and only if for every k the set Σ_k contains some negative assignment for every pair (Γ, t) where $\Gamma \varepsilon \mathfrak{G}$, and t is a repetition-free sequence of symbols from S_k of appropriate degree. This condition on the system (L_k, Σ_k) we will call *the condition F*. We may also give another formulation: (L_k, Σ_k) fulfills the condition F, if every set $\Sigma(\alpha_1, \ldots, \alpha_n)$ contained in Σ_k (where the symbols $\alpha_1, \ldots, \alpha_n \varepsilon S$) has a finite basis in (L, Σ). Our task is now to show that we can construct the successive extensions (L_k, Σ_k) so that each of them satisfies the condition F. To show this is enough to prove the theorem.

We will make the proof by induction. We first observe that the condition is fulfilled trivially by the system $(L_0, \Sigma_0) = (L, \Sigma)$. We assume then that (L_{k-1}, Σ_{k-1}) has been constructed so as to satisfy the condition F. We will show that it follows that (L_k, Σ_k) can be so constructed. (To show this for arbitrary k is now sufficient to prove the theorem.)

The system (L_k, Σ'_{k-1}) is formed as usual. We now assume an enumeration $p_1, p_2, \ldots$ of all pairs (Γ, t) where $\Gamma \varepsilon \mathfrak{G}$ and t is a sequence of appropriate degree from S_k. We want to form a series of corresponding negative assignments in the following way: N_1 is a negative assignment to p_1, which is consistent with Σ'_{k-1}, and in general N_m is a negative assignment to p_m, which is consistent with $\Sigma'_{k-1}, N_1, \ldots, N_{m-1}$. (If we have an enumeration of all the sentences in L_ω, we can take N_m as the "first" – – if any – – sentence which satisfies these conditions.) If we have such a series, we can form (L_k, S_k) in two steps: We first form a system (L_k, Σ''_{k-1}) by adding the set $\{N_m\}$ to Σ'_{k-1}. (This system must be consistent, since every finite subset of $\{N_m\}$ is consistent with Σ'_{k-1}.) We then construct (L_k, Σ_k) as a complete consistent extension of (L_k, Σ''_{k-1}). This system clearly satisfies the condition F.

The only thing, which now remains to be proved, is that under the assumptions made we can always find such a complete series $N_1, N_2, \ldots$ of negative assignments, which is consistent with Σ'_{k-1}. To prove this, we assume that there are such assignments $N_1, \ldots, N_{m-1}$, but that it is not possible to add a negative assign-

ment of p_m without introducing a contradiction. If $p_m = (\Gamma, (\alpha_1, \ldots, \alpha_n))$, where $\Gamma = \{A_i(x_1, \ldots, x_n)\}$ this means that the set $\{A_i(\alpha_1, \ldots, \alpha_n)\}$ is deducible from $\Sigma'_{k-1}, N_1, \ldots, N_{m-1}$. If we let $b_1, b_2, \ldots$ be the symbols of s_k, this can be written:

$$(1) \qquad \Sigma_{k-1}, \{Q_r(b_r)\}, B \rightarrow \{A_i(\alpha_1, \ldots, \alpha_n)\}$$

where the statements $Q_r(b_r)$ are the statements of Σ'_{k-1}, which correspond to existential statements $(Ex)Q_r(x)$ in Σ_{k-1}, and B is the conjunction of the statements $N_1, \ldots, N_{m-1}$.

We will show that this implies that the set $\{A_i(x_1, \ldots, x_n)\}$ has a finite basis, and thus that our assumptions are impossible.

It is important to observe in relation (1), that the left-hand side is consistent, that Σ_{k-1} contains no symbols from s_k, each sentence $Q_r(b_r)$ contains no symbol from s_k except b_r, and the set $\{A_i(\alpha_1, \ldots, \alpha_n)\}$ contains no symbols from S except $\alpha_1, \ldots, \alpha_n$, some of which may belong to s_k and some to S_{k-1}. We see now that we may apply Lemma 1 to effect some simplifications in the relation (1). Every sentence $Q_r(b_r)$, such that b_r does not occur anywhere else, may be dropped, – – since it may be replaced by the corresponding existential statement, and this is already in Σ_{k-1}. This means that all except a finite number of the sentences $Q_r(b_r)$ may be dropped. We write the remaining ones in a conjunction together with B, and we have a relation:

$$(2) \qquad \Sigma_{k-1}, C \rightarrow \{A_i(\alpha_1, \ldots, \alpha_n)\}$$

where the left-hand side is still consistent.

Let $\beta_1, \ldots, \beta_q$ be those symbols of S_{k-1} which occur in C or in $\{A_i(\alpha_1, \ldots, \alpha_n)\}$. (There can be only a finite number of them.) By again applying Lemma 1 and observing that (L_{k-1}, Σ_{k-1}) is complete, we see that we can, in the relation (2), dispense with all sentences of Σ_{k-1} which contain symbols of S_{k-1} other than $\beta_1, \ldots, \beta_q$. But the remaining subset of Σ_{k-1} is $\Sigma(\beta_1, \ldots, \beta_q)$. We write this result:

$$(3) \qquad \Sigma(\beta_1, \ldots, \beta_q), C \rightarrow \{A_i(\alpha_1, \ldots, \alpha_n)\}$$

But we know that (L_{k-1}, Σ_{k-1}) satisfies the condition F, and this means that $\Sigma(\beta_1, \ldots, \beta_q)$ must have a finite basis in $(L,$

Σ). If we let D be a basis we have $\Sigma, D \rightarrow \Sigma\ (\beta_1, \ldots, \beta_q)$, and (3) can be replaced by:

(4) $$\Sigma, C, D \rightarrow \{A_i(\alpha_1, \ldots, \alpha_n\}$$

If here we conjoin C and D and eliminate all constants except $\alpha_1, \ldots, \alpha_n$, we get:

(5) $$\Sigma_0, E(\alpha_1, \ldots, \alpha_n) \rightarrow \{A_i(\alpha_1, \ldots, \alpha_n)\}$$

From this it follows that the set $\Gamma = \{A_i(x, \ldots, x_n)\}$ has a finite basis, which is contrary to our assumption. With this we have shown that the desired series $N_1, N_2, \ldots$ of negative assignments exists. This completes the proof of the theorem.

As an obvious consequence of T 7 and T 8 we now get the desired converse of T 6:

T 9. Let (L, Σ) be a complete consistent system. If some maximal consistent condition-set in this system is without finite basis, then the system is not $\aleph_0$-categorical.

The Boolean algebra $B(\Sigma, x_1, x_2, \ldots, x_n)$

By $L(x_1, \ldots, x_n)$ we will denote the class of those formulas in L which contain no free variables except $x_1, \ldots, x_n$. Given some class Σ of sentences of L, the formulas of $L(x_1, \ldots, x_n)$ form a Boolean algebra, if we regard conjunction as intersection, disjunction as sum, negation as complement, and consider two formulas A and B identical, if $A \equiv B$ is a consequence of Σ [4]. This Boolean algebra we designate $B(\Sigma, x_1, \ldots, x_n)$. The maximal consistent condition-sets in (L, Σ) in the variables $x_1, \ldots, x_n$ are the maximal (dual) ideals in this Boolean algebra. The maximal consistent condition-sets with finite basis are principal ideals in $B(\Sigma, x_1, \ldots, x_n)$.

T 10. (Theorem of Ryll-Nardzewski). Let (L, Σ) be a complete consistent system. The following two statements about (L, Σ) are then equivalent:

(a) (L, Σ) is $\aleph_0$-categorical.

(b) For every n, the algebra $B(\Sigma, x_1, \ldots, x_n)$ is finite,

Proof. Assume that (b) does not hold, i.e. for some n, $B(\Sigma, x_1, \ldots, x_n)$ is infinite. It is known that every infinite Boolean algebra has some non-principal maximal ideal. Translated to the terminology we have used, this means that there is some maximal consistent condition-set in (L, Σ) without finite basis. By the preceding theorem it follows that (L, Σ) is not $\aleph_0$-categorical. To prove the implication the other way we assume that (b) holds. It follows that all maximal ideals are principal ideals, i.e. all maximal consistent condition-sets in (L, Σ) have a finite basis. Statement (a) follows then by *T* 6.

We may restate essentially the same result in terms of structures.

Def. If $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ are two sequences of elements in an L-structure M, we say that they *have the same type* in M if they determine the same maximal consistent condition-set.

T 11. Given an L-structure M, the corresponding complete system (L, Σ_M) is $\aleph_0$-categorical if and only if for every n the number of different types of n-sequences in M is finite.

Proof. Assume that there are m different types of n-sequences. If $A(x_1, \ldots, x_n)$ and $B(x_1, \ldots, x_n)$ are two conditions which are satisfied by the same types of sequences, then

$$A(x_1, \ldots, x_n) \equiv B(x_1, \ldots, x_n)$$

holds generally for M and is therefore a consequence of Σ_M. It follows that there can be at most 2^m non-equivalent conditions in $(x_1, \ldots, x_n)$, i.e. that $B(\Sigma_M, x_1, \ldots, x_n)$ is finite. Since this holds for every n, (L, Σ_M) is $\aleph_0$-categorical according to the preceding theorem. The converse is immediate.

This theorem gives an easy test to decide for any given M whether (L, Σ_M) is $\aleph_0$-categorical. Take, for instance, M to be the structure of natural numbers in terms of the successor relation. Obviously each element has a different type, i.e. there are infinitely many types of 1-sequences. It follows by the theorem that (L, Σ_M) is not $\aleph_0$-categorical.

IV. *Summary*

We have considered complete consistent systems in the first-oder predicate calculus with identity, and have studied the set of the models of such a system by means of the *maximal consistent condition-sets* associated with the system. The results may be summarized thus: (a) A complete consistent system is $\aleph_0$-categorical (=categorical in the denumerable domain) if and only if for every n, the number of different conditions in n variables is finite (T 10). (b) If a complete consistent system has a model M with finite character (i.e. a model M such that every maximal consistent condition-set satisfied in M has a finite basis), then this model M is uniquely characterized by the property that every other model is an arithmetic extension of M (T 5). (c) Every complete consistent system, which has only a denumerable number of different associated maximal consistent condition-sets, has a model with finite character (T 8).

REFERENCES

1. Henkin, L. The completeness of the first-order functional calculus. *The Journal of Symbolic Logic, 14, 1949.*
2. Los, J. The algebraic treatment of the methodology of elementary deductive systems. *Studia Logica II, 1955.*
3. Mostowski, A. Quelques observations sur l'usage des méthodes non finitistes dans la méta-mathématique. *Colloques Internationaux du C.N. R.S., 1958.*
4. Tarski, A. Foundations of the Calculus of Systems. Reprinted in *Logic Semantics, Metamathematics, 1956.*
5. Tarski, A., Vaught, R. L. Arithmetical extensions of relational systems. *Compositio Mathematica, 13, 1957.*

Aseity and Dependence in Leibniz's Metaphysics

by

ROBERT G. TURNBULL

(State University of Iowa)

In a letter to Nicolas Remond (1715), Leibniz wrote:

> If someone were to reduce Plato to a system, he would render a great service to mankind, and *it would then be clear that my own views approach his somewhat.*[1]

I propose to take this remark of Leibniz quite seriously. My point in so doing is not, however, an historical one in the sense that I hope to show that the reading of Plato influenced Leibniz to write as he did, though Leibniz himself insisted that he was influenced by Plato. Nor is my aim merely the making of a comparison. It is, rather, systematic and, as such, both analytical and critical: analytical, in that I hope to use the fundamental features of Plato's metaphysics, with certain of its inherent problems, as an aid in descrying the structure of Leibniz's metaphysics; critical, in that I hope such limning to exhibit rather clearly certain "breaking points" in the Leibnizian system. This approach to the understanding and criticism of Leibniz is, so far as I know, a novel one,[2] and will, by implication, involve criticism of some current interpretations of Leibniz.

It is necessary to state, first, the fundamental features of Plato's metaphysics. This cannot be described as the "systematizing" which Leibniz felt Plato "deserved". It is, however, a *step* towards that systematizing. In the course of the paper I wish to

[1] G. W. Leibniz, *Philosophical Papers and Letters,* translated and edited by Leroy E. Loemker (Two vols., Chicago, 1956) – hereafter referred to as "Loemker" – Vol. II, p. 1072. Italics are mine.

[2] Though almost every Leibniz interpreter has duly noted the more obvious similarities between him and Plato. The attempt here is to cut somewhat more deeply than any of these.

show that a serious attempt at systematizing Plato, involving, as it must, ridding the Platonic pattern of paradox and inconsistency, moves one well along towards Leibniz's own system. Since my main concern is with Leibniz, I shall offer no argument for my interpretation of Plato, nor do I wish to claim that it takes account of all the diverse elements in Plato's writings. The defense of the interpretation as doing justice to the main drift of Plato's metaphysics has been made in another paper.[3]

Metaphysical bed-rock in Plato consists of *souls* and *forms*. Otherwise stated, the being of souls and forms is, as certain medievals had it, *a se*. The Platonic metaphysical inventory is not exhausted, however, by being *a se*, but (to continue in the medieval vocabulary) includes being *per aliud* (or *dependent* being). What have being *per aliud* are, as Cornford has called them, *"immanent characters"*.[4]

Souls are intrinsically active; their *esse* is *agere*. (As the *Phaedrus* has it, they are "self-movers"; as the *Phaedo* has it, "strivers".) Forms are static, unchanging and unchanged. Qua objects of the strivings of souls, they provide, together with souls, the rationale for the coming-to-be and passing away of immanent characters. (There is here, obviously, an historical root of the notion of immanent, as opposed to transeunt, causality.)

Immanent characters are dependent, or *per aliud*, in three different ways. First, they are dependent *per participationem*, as *the tall in Phaedo* (an immanent character) is said to "participate" in *the tall itself* (a form). This is, of course, a one-many matter, there being (or being possible) many *talls*, but only one *tall itself*. Second, immanent characters are dependent *per praesentiam, i.e.*, as being "in" something else. The stock Platonic phrase, 'the f in x', carries the obvious requirement that there is no f without their being an x for f to be "in". Immanent characters are dependent *per praesentiam* upon souls. Third, immanent characters are dependent *per actum, i.e.*, they depend on

---

[3] "Aristotle's Debt to the 'Natural Philosophy' of the *Phaedo*", *Philosophical Quarterly*, April, 1958, pp. 131–143.

[4] F. M. Cornford, *Plato and Parmenides* (London, 1939), p. 78.

the "moving" of something else. They are in this respect dependent upon souls *qua* strivers.

The Platonic metaphysical pattern can be thought of as the result of reflection upon change. Change or, if you will, "becoming" is a fact. There "is" the flux of immanent characters. But change cannot be an ultimate fact. What change requires are a changer (or changers), objects (or objectives) for changers to strive for, a being (or beings) for changes to "participate" in, and, finally, a being (or beings) for changes to be "in". Hence, souls and forms are necessary to explain change. If, furthermore, there is any ordering of change, there must be a structuring of forms to make possible "rational" striving on the part of souls (at any rate, of some souls sometimes).

When the basic metaphysical pattern of Plato is so simply stated, it fairly bristles with difficulties, some of which have long since been exposed. It has, nevertheless, the merit of being rather simple and invites emendations which preserve its character as a metaphysics of aseity and dependence. It is part of my intention in this paper to show that the fundamental features of Leibniz's metaphysics can be properly thought of as emerging from an attempt to make appropriate emendations in the Platonic pattern.

There is a patent similarity between Platonic souls and Leibnizien "metaphysical points"; both are such that their *esse* is *agere*. This comment, at the moment, is not really very illuminating. It wants exploitation. To exploit it will require our taking a rather circuitous route. We start naturally, I believe, with some critical reflections upon the Platonic doctrine of dependence *per praesentiam*.

Consider the notion that immanent characters are "in" souls as a result (*per actum*) of the strivings of souls after forms. Suppose soul a to be striving after form ϕ and immanent character f_1 (one which participates in ϕ)[5] to be in a as a result of that striving. Make similar suppositions for b, c, d, and so on. It is immediately evident that, if nothing further is to be said on the

[5] An immanent character is, of course, a kind of "particular". I have, therefore, used numerical subscripts to distinguish different immanent characters which participate in the same form.

point, Platonic souls are, in an appropriate Leibnizian sense, "windowless". But, in the pattern of Plato's metaphysics, nothing further is relevant to the point. In one sense, souls *qua* striving to attain forms are causes ("efficient"). In another, forms *qua* objects of the strivings of souls are causes ("final"). In yet another, forms *qua* participated in by immanent characters are causes ("formal"). In *no* sense, however, are there causes which involve interconnection or interrelation between souls. Indeed, in the Platonic pattern, causal interconnection between souls would be a contradiction, for the being of souls is *agere*, not *pati*. It is of at least passing interest that one of the chief evidences in Plato for taking the *agere* of souls to be "striving to attain" the forms is a favorite passage of Leibniz's. Leibniz wrote, for example:

> There is an admirable passage in Plato's *Phaedo* in which he justly blames Aanaxagoras for the same thing which displeases me in Descartes.[6]

It follows that the Platonic "world of becoming" is, as it were, "located in" souls. (It matters not that Plato would, most likely, have been uneasy about this conclusion; it does follow from the pattern.) It is difficult, however, to understand how immanent characters might be "in" souls. We must be careful here to avoid a possible confusion. The difficulty is *not* that concerning how one and the same characteristic can characterize many particulars or individuals. In the question of immanent characters' being "in" souls there is no vestige of a one many problem. To avoid this confusion, one might even say that immanent characters are particulars and that the opposites, *a se* and *per aliud* (where *per praesentiam* is concerned) do not have any appropriate resemblance to the opposites, abstract and concrete. So successful has been the modern attack on realism (*in the medieval sense*) that it has not been easy even for professional historians of philosophy to comprehend that a whole philosophical tradition (what I am

[6] Loemker, Vol. I, p. 419. See also Loemker, Vol. I, pp. 486–87 and, especially, pp. 542–43. For the bearing of the *Phaedo* passage on my interpretation of Plato's metaphysics, see "Aristotle's Debt to the 'Natural Philosophy' of the *Phaedo*", *loc. cit.*, pp. 134–135.

calling the "metaphysics of aseity and dependence") took certain non-abstract existents as dependent. Unfortunately, one searches through Plato's writings in vain if he is looking for a solution to our problem, largely, I suppose, because his "metaphysical pattern" was never rendered explicit.

Suppose that '*a* if f_1' is a sentence, where *a* is a soul and f_1 an immanent character. In this case, if one were casting about for a means of rendering intelligible the manner in which immanent characters may be "in" souls, he might find the dictum, '*Praedicatum inest subjecto*', worthy of his attention. What the dictum *suggests* is that, in our example, f_1 is "in" *a* somewhat as, in a Moorean sense, *ma'e* is "in" *brother* or *straight line* is "in" *triangle*. Unless we add another dictum, namely, 'One soul, one immanent character', this has the unfortunate consequence of requiring souls to be *complex*. But neither Platonic souls nor Leibnizian metaphysical points can be complex. If we add the second dictum, it will be impossible for more than one immanent character to be "in" a soul, even successively (in time). And this, too, is inconsistent with the Platonic and Leibnizian doctrines.

Faced with this dilemma, is there any possibility that the dictum, '*Praedicatum inest subjecto*', may still help in explaining how an immanent character may be "in" a soul? There is one possibility. It rests upon there being two ways in which an immanent character is dependent upon a soul, namely, *per praesentiam* and *per actum*. With respect to the *per actum* "relation" between a soul and an immanent character which is "in" it, we may wish to say that the soul *does not* stand to the immanent character as relatively complex to relatively simple, but that it *does* with respect to the *per praesentiam* "relation". (One is reminded of the Bruno–Spinoza distinction between *natura naturans* and *natura naturata*.) This solution to our dilemma, resting upon a tenuous distinction between "aspects" of souls, is scarcely a satisfactory one. It points, nevertheless, in the direction of Leibniz's solution, though we must examine some other matters before trying to present Leibniz's doctrine. I turn, therefore, to another line of development.

Consider now that, given our presuppositions, if a certain im-

manent character, say f_1, exists, then there must be a soul for f_1 to be "in", for there are no "dangling" immanent characters. (Cf. Leibniz's comment: "Accidents cannot be detached from substances and march about outside of substance, as the sensible species of the Scholastics once did.")[7] It follows equally well from our presuppositions that, if f_1 exists, then there must be a form for f_1 to participate in. We seem, therefore, to have two bridges from dependent being to being *a se*. For immanent characters are dependent both *per praesentiam* and *per participationem* (ignoring for the present purpose *per actum*). Our Platonic riches are well-nigh embarrassing.

One way of exhibiting a source of that embarrassment is to advert to logical considerations. One may say (as we did a short time ago) that a certain soul, *a*, if f_1. One may also say that f_1 is ϕ (a certain form). From the fact that these locutions reflect, respectively, *per praesentiam* and *per participationem*, we may wish to say that the first 'is' means 'has in it' (though, of course, we don't understanding the way immanent characters can be "in" souls) and that the second 'is' means 'participates in'. Taking this distinction to be a logical feature of Platonism, one must say that the two sorts of sentences are irreducible and that there are, in the Platonic logic, two (at least) irreducible meaning of 'is'. And this fact is, of course, connected with the Platonic metaphysics having two kinds of beings which have aseity.

It is necessary to contrast either or both of the Platonic meanings of 'is' with certain options which are of greatest contemporary appeal. The contemporary logical nominalist, holding (in the terms used in this paper) that there is only one sort of being *a se* and *no* dependent being, holds in a sense, that 'is' has no meaning.[8] 'In a sense' merely indicates his denial of what is commonly called "predication". The contemporary logical realist holds that there are two sorts of beings with aseity and *no* dependent

[7] *Monadology*, 7, in Loemker, Vol. II, p. 1045.

[8] In the required sense there is no distinction between nominalism and what has been called "logical idealism". For, put linguistically, the nominalist says that all one needs are zero-level descriptive terms, the idealist, first-level terms.

being, the 'is' being taken to express the "relationship" between these two sorts of being. Contemporary nominalists and realists alike reject what I have called the "metaphysics of aseity and dependence"; they accept no meaning of 'is' which is taken to reflect a "relation" between what is *a se* and what is dependent. There is, therefore, nothing like contemporary realism in the Platonic scheme, nor is there, I shall urge, in the Leibnizian. I have been ignoring dependence *per actum,* even though reconciling the 'is' corresponding to it would fit well with the paradox arising from our reflections on how an immanent character can be "in" a soul. As I wish now to make certain comments about Aristotle, I think some expository advantage is gained from ignoring it for the moment.

Aristotle's meanings of 'is' as explained in the *Categories,* namely, 'present in' and 'predicated of', are *superficially* similar to the Platonic ones.[9] The appearance of similarity is enhanced by the Aristotelian distinction between so-called common and proper names, by means of which 'predicated of' may appear to play the role of the Platonic 'is' of participation. According to it, something may name commonly what 'f_1' names properly. Indeed, something may name commonly what '*a*' names properly. This extension is absurd if the common-proper names distinction is taken to reflect the metaphysical "relation" of participation. *There are no soul-forms in the Platonic scheme.* That Aristotle did not construe his distinction as reflecting "participation" is a measure of the philosophical triviality of the common-proper names distinction. Participation, naturally enough, virtually disappears in the serious Aristotle, surviving slightly in the doctrine that there are real similarities between his substances which justify causal inferences or explanations. (This watering-down of participation is, at root, the introduction of transeunt causality and, so far forth, the denial of the metaphysics of aseity and dependence. In this sense, the celebrated "empiricism" of Aristotle leads straight to Hume.)

[9] See my "Aristotle's Debt to the 'Natural Philosophy' of the *Phaedo*", *loc. cit.*, pp. 140 ff. for an attempt to explain how Aristotle was led to think that his distinction would do the work of Plato's.

What correspond to Platonic souls in Aristotle are substances, to immanent characters, accidents. But, rejecting the forms (though attempting to keep a metaphysics of aseity and dependence), Aristotle complicated substances. *Qua* "form", a substance is a nature, an hypostatized tendency to have in the substances certain kinds of immanent characters (accidents). *Qua* "matter", a substance is a complex capability of variation with respect to immanent characters' being "in" it. There is here some resolution of our dilemma concerning immanent characters' being "in" souls, the resolution coming (as in our very tentative suggestion) in taking substances as at once simple and complex. In Aristotelian "form", however, there seems admixture of the roles of both *per actum* and *per participationem*.[10] The role of *per praesentiam* is taken – roughly – by "matter".

I shall ignore the puzzles of hylomorphism which derive from taking as simples compounds of non-existents. What struck Leibniz as especially disturbing in the "Peripatetics"[11] comes about because of the dilution of *participatio*. On the one hand is the Aristotelian tendency to allow "free" immanent characters (accidents) with no participational support for their dependence (though leaving out such support in a metaphysics of aseity and dependence is *strict nonsense*). On the other hand is the much-derided tendency to postulate a "faculty" to account for actually observed immanent characters, thus encouraging the production of explanations which do not explain, as in the ubiquitous ex-

---

[10] I believe that a case can be made for Aristotle's taking change, *qua actus*, as derivative from form and matter, thus eliminating *actus* completely from what I have called being *a se*. The issue is a complex one, and, fortunately, deciding the matter is not necessary for the present purpose.

[11] Examples: "... with good reason I despised the method of those who use only forms or faculties of which nothing is understood" (Loemker, Vol. II, p. 740); "... the Aristotelians, especially the Scholastics, cared more about asking questions than about answering them" (Loemker, Vol. II, p. 708); "The scholastics employed inaptly a general notion, when they used it 'form' to explain particular phenomena ... when the arrangement of species is the question, it is useless to dispute about the substantial forms" (*New Essays Concerning* Human Understanding, A. G. Langley translation. La Salle, Ill., 1949 pp. 347 and 349).

ample of the "dormitive power" of opium. The production of such "explanations" derives from mixing immanent and transeunt causality and is, patently, the price of diluting (or abandoning) participation in a metaphysics of aseity and dependence.

Aristotle's attempt to simplify the Platonic scheme could not have succeeded, for he kept the two Platonic meanings of 'is' (after a fashion) and, in effect, abandoned participation. The attempt is rather like that of the contemporary nominalist to keep predication after excising one of the realist's kinds of being. As we saw earlier, nevertheless, it seems eminently desirable to simplify Plato's metaphysical pattern, for assigning aseity to both souls and forms is, to say the least, embarrassing. It was part of the genius of Aristotle to have seen the need for simplification. What Leibniz saw was the failure of Aristotle's attempt and the need, in a plausible metaphysics of aseity and dependence, to accompany the metaphysical simplification (of Plato's system) with the preservation of the Platonic meanings of 'is'. Interestingly enough, the Platonic meanings (including the force of one corresponding to dependence *per actum*)[12] are packed into *one* Leibnizian meaning of 'is', *viz.*, that involved in the dictum, '*Praedicatum inest subjecto*'. It is of the utmost importance to keep in mind that the 'is' here is *not* the predicational 'is' of the contemporary realist. The following passage from the correspondence with Arnauld exhibits the inclusion of the Platonic meanings into the one Leibnizian meaning.

---

[12] Cf. "The Correction of Metaphysics", in Loemker, Vol. II, p. 709: "I will say for the present that the concept of *forces* or *powers*, which the Germans call *Kraft* and the French *la force*, and for whose explanation I have set up a distinct science of *dynamics*, brings the strongest light to bear upon our understanding of the true concept of *substance*. Active force differs from the mere power familiar to the Schools, for the active power or faculty of the Scholastics is nothing but a close [propinqua] possibility of acting, which needs an external excitation or a stimulus, as it were to be transferred into action. Active force, in contrast, contains a certain action or entelechy and is thus midway between a faculty of acting and the action itself and involves a conatus. It is thus carried into action by itself and needs no help but only the removal of an impediment."

> . . . always, in every true affirmative proposition, whether necessary or contingent, universal or particular, the notion of the predicate is in some way included in that of the subject. *Praedicatum inest subjecto;* otherwise I do not know what truth is. Now I do not demand any further conection here than what is really found [*a parte rei*] between the terms of a true proposition, and it is only in this sense that I say that the concept of an individual substance includes all its events and all its denominations, even those which are commonly called extrinsic, that is, those which pertain to it only by virtue of the general connection of things and from the fact that it expresses the whole universe in its own way. For *there must always be some foundation for the connection between the terms of a proposition, and this must be found in their concepts.* This is my great principle, with which I believe all philosophers should agree, and one of whose corollaries is the commonly held axiom that nothing happens without a reason, which can always be given, why the thing has happened as it did rather than in another way, even though this reason often inclines without necessitating. A perfect indifference is a deceptive or incomplete assumption.[13]

Dependence *per actum* is suggested by Leibniz's taking the principle of sufficient reason as a "corollary" of his "great principle". Dependence *per participationem* is found in the insistence on the "windowlessness" of the substances and in sufficient reason's requirement that a "thing" happen this way rather than in another way (and in its being an explanation for such happening). Dependence *per praesentiam* is somehow to be found in the "connection between the terms of a proposition". I turn from this to considering some points of detail.

Let us start from being *a se*. Consider the following sentence-form: '— — — — is — — — —'. Let the blank to the left of 'is' be called the "aseity place", that to the right of 'is', the "dependence place". Suppose the blanks filled in such a manner that the resulting sentence is true. Let '*A*', e.g., be in the aseity place. *A*, then, has some of the features of a Platonic soul or an Aristotelian substance; in particular, it cannot be a predicate, if to be a predicate is, *ipso facto,* to be dependent. What appears in the de-

[13] Loemker, Vol. I, p. 517.

pendence place must, moreover, name something which is dependent *per actum* upon *A*. *A* must also be similar to the Platonic forms in that what is named in the dependence place must be dependent *per participationem* upon *A*.

Let us now make a supposition derived from Plato's *Sophist*, namely, that the fundamentum of *not* is *different from* and that this is to be understood in terms of participation. If, with this in mind, we attempt to think that *B*, e.g., is not *A* and that '*B*' might properly appear in the aseity place, we find the attempt abortive. If *B* is not *A*, then both *B* and *A* must be dependent. If *B* and *A* are *a se*, then '*B* is not *A*' is nonsense. It follows that what is named in the aseity place in our sentence "is without limits, without any negation, and consequently without any contradiction".[14] It also follows that it has "nothing outside of it which is independent of it".[15] There is thus only one being which is *a se*. These last statements must, of course, be (in the sense of the "first" Wittgenstein) heuristic nonsense or else "ordinary language" statements about a Leibnizian "ideal language schema" – unless we are to introduce paradoxes all over again.

What goes in the dependence place in our sentence-form must, obviously, be the name of something dependent *per actum, per participationem,* and *per praesentiam* upon being *a se*. As a limiting case, what is so named is all possible worlds, including the actual one. Here is the root of the doctrine of "continuous creation" in a metaphysics of aseity and dependence; and it is a simple consequence of the doctrine of "predication" in such a metaphysics and of what I have called the "simplification" of the Platonic pattern.

The notion of possible worlds is inevitably linked with the name of Leibniz, but rendering it intelligible in a metaphysics of aseity and dependence is quite another matter. Closely tied with it are two vexing issues, both of which bring us back to Plato. The first concerns the intelligibility of the notion of *dependent* substances. It is quite clear that Leibniz thought there were de-

---

[14] *Monadology* 45, in Loemker, Vol. II, p. 1051.
[15] *Monadology* 40, in Loemker, Vol. II, p. 1050.

pendent substances and that such dependence could be made intelligible. In Plato, it must be noted, *souls are not dependent*; the dependent for him is limited to immanent characters. Even Aristotle's doctrine of the "four causes" seems designed to account for only the coming-to-be of immanent characters (accidents) and gives the impression of *ad hoc* improvisation when applied to primary substances. Spinoza, whose metaphysics of aseity and dependence is somewhat like Leibniz's, regarded the notion of a dependent substance as unintelligible. That he thought so was linked in Leibniz's mind with the Spinozistic claim that all possibilities are – at least from God's point of view – actual.[16] The second vexing issue arises from the fact that nothing in the Platonic pattern explains any *ordering* in the strivings of any individual soul and, consequently, in the series of immanent characters in that soul. Plato several times suggested that the structure of the world of forms may function as a pattern for sequences of immanent characters in souls, as, *e.g.*, the expectation that the guardians of the *Republic* will "imitate" the ordering of the forms (after apprehending the "Idea of the God") in the ideal state. Unfortunately there is little to go on in these suggestions. One might say that there is a "felt need" in a metaphysics of aseity and dependence for there to be an ordering in dependence which has its source in aseity. After all, the reason for distinguishing kinds of dependence will scarcely allow one to stop short of accounting for everything concerning dependent being from being *a se*.

Leibniz clearly takes '*Praedicatum inest subjecto*' to apply equally to *dependent* substances and to substance with aseity.

[16] Cf. letter to Christian Philipp, 1680, in Loemker, Vol. I, p. 420: "... the opinions which Spinoza has expounded more clearly, namely, that justice, beauty, and order are things merely relative to us but that the perfection of God consists in that magnitude of his activity by virtue of which nothing is possible or conceivable which he does not actually produce." Cf. also letter to Louis Bourguet, 1714, in Loemker, Vol. II, p. 1077: "according to Spinoza ... there is only one substance. He would be right if there were no monads; then everything except God would be of a passing nature and would vanish into simple accidents or modifications, since there would be no substantial foundation in things, such as consists in the existence of monads."

But how is this to be possible? Suppose, *e.g.*, that a certain determination, f_1, is "in" a certain dependent substance, *a*. If we try, in the framework of dependence predication, to say '*a* is f_1', it is immediately evident that '*a*', being the name of a *dependent* substance, cannot properly occupy the aseity place. Our question becomes: How can one give simultaneous expression *both* to the dependence of *a* and to the dependence of f_1 on *a*? There is only one possible way, and that requires treating *a*'s being f_1 (or, if you will, f_1's being in *a*) as itself dependent. '*a* is f_1' would then have to be regarded as either misbegotten (if *a* really is dependent) or as elliptical for something like '(being *a se*) is (*a*'s being f_1)', where the 'is' is, of course, that of dependence predication. Nor is this a surprising development. Since both souls and forms are, as it were, collapsed together in Leibnizian being *a se*, though their roles are retained, we should expect, if dependent substances enter the picture at all, a *double* application of the various kinds of dependence.

When the matter is so stated, what I am attributing to Leibniz has a striking resemblance to a doctrine of C. I. Lewis.

> . . . any participial phrase like 'Mary making pies now' can have either of two senses. In one of these it is a predicable expression, like the adjective 'hot' or 'sweet': in the other it is abstract and pronomial, like 'hotness' or 'sweetness'. It is the former of these in which it is equivalent to 'that Mary is making pies' and is to be identified with the proposition. It is in this sense that it is predicable of a world. In the other -- the abstract sense – it *names* the state of affairs attributed to the actual world by asserting "Mary is making pies".[17]

Though my point in quoting Lewis here is more illustrative than systematic, I should like it noted that making the distinction between intension and extension *with respect to sentences* links one with the metaphysics of aseity and dependence. For, whether the extension of a sentence be a "truth value" or "reality" (or, of course, the "null class"), the intension which is

[17] C. I. Lewis, *An Analysis of Knowledge and Valuation* (La Salle, Ill., 1946), pp. 52–53.

(somehow) "predicated of" it has a complexity which cannot be reconciled with the aseity of both subject and predicate in what I have called "contemporary realistic logic". But this is material for another paper.

Suppose that we took '*a*'s being f_1 'in Lewis' second sense, *i.e.*, as "abstract and pronomial"; it would then not be unreasonable to speak of what it names as an *essence* or as a *possibility*. It would, of course, be more than a "mere" essence or possibility, if it could be truly predicated of being *a se*; but, even so, it would be more only *qua* truly predicated. Leibniz favored the linkage of essence and possibility. He wrote, *e.g.*, in "The Radical Origination of Things":

> And just as *possibility is the principle of essence*, so perfection or degree of essence is the principle of existence.[18]

The latter part of this quotation will enter our exposition a little later. It will serve now, however, to give a hint of the direction of my thought.

So far we have spoken only of what I shall call *accidental* or *characteral essences* (or possibilities). Since Leibniz took seriously the notion of dependent substances *qua* continuants, it is evident that we shall have to contrast characteral essences with *substantial essences* (or possibilities). This is, of course, linked with one of the most familiar notions in Leibniz's philosophy.

> . . . we can say it is the nature of an individual substance or complete being to have a concept so complete that it is sufficient to make us understand and deduce from it all the predicates of the subject to which the concept is attributed. An accident, on the other hand, is a being whose concept does not include everything that can be attributed to the subject to which the concept is attributed. Thus the quality of king which belonged to Alexander the Great, if we abstract it from its subject, is not determined enough to define an individual, for it does not include the other qualities of the same subject or everything which the concept of this prince incoudes. God, on the contrary, in seeing the individual notion or "haecceity" of Alexander, sees

[18] Loemker, Vol. II, p. 793. Italics are mine.

> in it at the same time the basis and reason for all the predicates which can truly be affirmed of him – for example, that he will conquer Darius and Porus – even knowing a priori (and not by experience) what we can know only through history – whether he dies a natural death or by poison.[19]

Suppose that we use 'n_1', 'n_2', etc. ("natures") to refer to haecceities or individual concepts. How, then, might one refer to substantial essences? The question is a difficult one, though it is evident that individual concepts and substantial essences are linked.

One thing is quite certain: Leibniz took every existent dependent substance to be *simp'e*. It is equally certain that every merely possible dependent substance is also to be taken as simple, for Leibniz quite obviously does not take the conferring of existence upon a possible substance (or substantial essence) as the adding, as it were, of simplicity to what was, *qua* possible, complex. Individual concepts themselves, however, are obviously complex in that each "contains" all the determinations (and in order) of the substance of which it is the concept. Finally, it seems obviously necessary that substantial essences be such that, in the sense of '*Praedicatum inest subjecto*', they have "in" them characterial essences. The solution is suggested by the account given above of characteral essences. If, in our example, one refers to a certain characteral essence by '*a*'s being f_1', then the reference to the substantial essence should be by '*a*'s being n_1'.

If we take '*a*'s being n_1' in both of Lewis' senses, it seems possible to preserve the dependence of substances (or substantial essences) and to suggest how determinations of substances (characteral essences) can be "in" them. *a*'s being f_1 would be, on this accounting, doubly derivative, as being in *a*'s being n_1, the latter, in turn, being in being *a se*. That '*a*' in our example occurs in expressions for both substantial essence and for the characteral essences which are in (now, at last, without quotes) that substantial essence reflects, in part, Leibniz's insistence on the simplicity of substances and, in part, his equal insistence that "what

---

[19] "Discourse on Metaphysics", in Loemker, Vol. I, p. 472.

it is to be in or to inhere in a subject" is to be "a mode or state of a subject".[20] n_1 or f_1, apart from their being in *a*, would be equally "abstract". Furthermore, our exemplary locution, '*a*'s being n_1', suggests the dual role of substantial essences, *i.e.*, as adjectival and dependently substantival.

The difference between essences which are merely possible and those which are actual may easily be indicated by the differences between participial phrases and sentences using the 'is' of Leibnizian aseity and dependence. Spelling this out will enable me to make a certain interesting point with respect to what Leibniz called the "reality in possibilities". '(being *a se*)'s being ((*a*'s being n_1)'s being (*a*'s being f_1))' may thus indicate how a possibility (indeed both a characteral and a substantial possibility) is in being *a se*. What this refers to, however, cannot be dependently predicated for, otherwise, being *a se* would lack aseity. The totality of non-contradictory participial phrases of this sort delineate everything possible and these possibles *qua* existing (Leibniz's word) "in a certain region of ideas, if I may so call it, namely, in God himself".[21] It has been noted that Leibniz shifted from an earlier phase in which he argued, 'If God is possible, then he exists', to a later one in which he argued, 'If there are possibles, God exists'.[22] It is at least arguable that, given the foregoing, this is merely a shift of emphasis; the impossibility of dependently predicating what our exemplary participial phrase refers to makes it a virtually indifferent matter whether we speak of the "existence" of possibles or of the possibility of being *a se*.

> It is also true that the source not only of existences but also of essences is in God, insofar as these essences are real or insofar as there is something real in possibility. This is because the understanding of God is the region of eternal truths or of the ideas upon which they depend and because without him there would be no reality in possibilities – not only nothing existent but also nothing possible.[23]

[20] Correspondence with Des Bosses, in Loemker, Vol. II, p. 987.

[21] "Radical Origination of Things", in Loemker, Vol. II, p. 793.

[22] Cf. the difference between "Discourse on Metaphysics" 1. (In Loemker, Vol. I, p. 465) and "Monadology" 44 and 45 (In Loemker, Vol. II, p. 1051).

[23] "Monadology" 43, in Loemker, Vol. II, p. 1051.

Thus Leibniz gives a metaphysical status to essences or possibilities. Since, as we shall see shortly, 'existence', as applied to the actual world, is a defined term in the Leibnizian pattern, it is rather strange to say 'God exists' ('Being *a se* exists') or 'Essences exist'. One is tempted to say that 'Being *a se* exists' means ' 'Being *a se*' has an intensional referent, but a queer one which cannot be dependently predicated'. Similarly, one is tempted to say that 'Essences or possibilities exist' means ' 'Essences' has an extensional referent, but a queer one in that its intensional referent *includes* it'. Queer as this sounds, it is not a surprising consequence of the collapsing together of Platonic souls and forms *and* attempting to provide a rationale for the ordering of immanent characters in what, in the simplified scheme, are dependent substances.

Leibniz thought of the substantial essences as organized in such a manner as to constitute a set of "possible worlds". Each such possible world is a group of substantial essences (with the characteral essences which are in them) which are said to be "compossible". No substantial essence belongs to more than one possible world. And, finally, every possible world is, as it were, the "contradictory" of every other, so that, if any one of them is actual, it is impossible that any of the others might be actual. Though I do not think this notion of possible worlds an intelligible one, I should like to assume that it is in order to complete the description of the Leibnizian pattern. To say '*a* exists' is to say '(*a*'s being *n*) belongs to the possible world which has the greatest number of substantial essences in it'. That Leibniz means 'existence' to be defined in terms of 'essence' is supported by a number of texts.[24] The point I wish to make in this connection, however, is not merely a textual one. What I wish to point out is what I think is Leibniz's chief contribution to the plausibility

[24] Examples: "... Possible things are those which do not imply a contradiction; actual things are nothing but the best of possibles, everything considered" (Loemker, Vol. II, p. 833); "And just as possibility is the principle of essence, so perfection or degree of essence is the principle of existence (since the degree of perfection determines the largest number of things that are compossible)" (Loemker, Vol. II, p. 793).

of a metaphysics of aseity and dependence, namely, that, though essence is the principle of existence, it is logically impossible (self-contradictory) for all essences to exist. This does not imply nevertheless, that it is logically impossible for any given essence or even possible world to exist. It is arguable that, at root, what provides this result is attempting to cope with the problem of the derivation of the *ordering* of characters or modifications of a dependent substance in a metaphysics of aseity and dependence.

With this I think I may rest my case. When it is laid out as has been done above, proceeding from problems inherent in the Platonic metaphysical pattern, the structure of Leibniz's metaphysics can easily be seen as an ingenious attempt to maintain the fundamental features of the Platonic scheme while making what seem to be some necessary emendations. That this was a self-conscious attempt to make Platonism plausible I very much doubt;[25] that it can easily be thought of as such an attempt is all I have wished to show. Leibniz's is a remarkably thoroughgoing effort to render plausible the metaphysics of aseity and dependence, including all the Platonic modes of dependence – *per participationem, per actum,* and *per praesentiam* – and simultaneously to do justice to our pre-philosophical conviction that not all possibilities are (or can be) actual. It remains to make a critical observation.

I do not believe that the notion of possible worlds can be rendered intelligible in the sense required for the Leibnizian scheme. Suppose that f_1 and g_3 are contraries. Then it cannot be the case that a's being f_1 and a's being g_3 are both characteral essences. But a's being n_1 may have in it both of these characteral essences, for Leibniz expressly recognized that the same substance may have contrary attributes *at different times*. It follows that, unless time be taken as absolute, any given substantial essence may contain a contradiction, and there are no possible worlds in Leibniz's sense. Leibniz must, therefore, either take time to be absolute

[25] In a letter to Michael Gottlieb Hansch (1707), wherein he discusses Plato's metaphysics in summary fashion, Leibniz gave an interpretation of Plato which is in some respects like, in others, unlike, the one I have given. Loemker, Vol. II, pp. 962–967.

or give up the notion that the same substance may have contrary attributes. But, if time is absolute, Leibniz's metaphysics of aseity and dependence must be false; for, in that event, God will not have, so to speak, a non-temporal "glimpse" of the world.[26] And, if the same substance may not have it in contrary attributes (at different times), it is impossible for Leibniz's substance metaphysics to save the appearances.

We may express the same difficulty in another manner. The crucial distinction between Leibniz and Spinoza is that the former has the notion of dependent substantial essences.[27] These must, to accomplish Leibniz's purpose, be organized into possible worlds, each of which is incompatible with every other. If not, then, acting from the notion that essence is the principle of existence, all possibilities will be actual. This is, of course, precisely what Leibniz reprehends in Spinoza.[28] But how may one render intelligible the incompatibility of various possible worlds? The chief source of difficulty lies in the fact that a given substantial essence may have in it contrary characteral essences. Again, unless absolute time be presupposed, every substantial essence may be self-contradictory. But, if absolute time is presupposed, our difficulty about God's "glimpse" follows.

It may be objected that we are to think of substantial essences after the manner of algebraic formulae for the generation of series of numbers,[29] where the numbers in the several series cor-

[26] Cf., *e.g.*, "But there is an infinite difference between our intellect and the divine, for God sees all things adequately and at once" (Loemker, Vol. II, p. 963).

[27] "He [Spinoza] would be right if there were no monads; then everything except God would be of a passing nature and would vanish into simple accidents or modifications, since there would be no substantial foundation in things, such as consists in the existence of monads", from a letter to Louis Bourguet, in Loemker, Vol. II, p. 1077.

[28] Loemker, commenting on Leibniz's "On the Ethics of Spinoza", writes, "This is the point to which many of Leibniz's comments have been leading; not all possibilities exist, as Spinoza holds", in Loemker, Vol. I, p. 571.

[29] That Leibniz so thought of the matter is evidenced in several places. See *e.g.*, Letter of January 21, 1704, to De Volder (in Loemker, Vol. II, pp. 870–871); Letter of March 23, 1690, to Arnauld (in Loemker, Vol. II, p. 599).

respond to characteral essences. We may then think of possible worlds after the manner of more complex algebraic formulae which, as it were, comprehend large sets of our more simple formulae and together comprehend all of the more simple formulae. Aside from the difficulties involved in the analogy of numbers to characteral essences and the threat to Leibniz's substantial individualism in the more complex formulae, one objection is damning. By Leibniz's own insistence,[30] the selection of the more complex formulae is purely arbitrary; in the limiting case, one more complex formula will comprehend all of the simpler ones. The upshot of this critical observation is a general objection to the metaphysics of aseity and dependence. If, leaving aside certain bits of detail, the Leibnizian and the Spinozistic versions of it are the most plausible ones, then it founders in the one case because of its inability to show how all possibilities are not actual and in the other because of its insistence that all possibilities are actual.

[30] Cf. paragraph 6. of the "Discourse on Metaphysics" (in Loemker, Vol. I, pp. 469–470), especially the following: "For, as concerns universal order, everything is in conformity with it. So true is this that not only does nothing happen in the world which is absolutely irregular but one cannot even imagine such an event. For let us assume that someone puts down a number of points on paper entirely at random, as do those who practice the ludicrous art of geomancy; I maintain that it is possible to find a geometric line whose law is constant and uniform and follows a certain rule which will pass through all these points and in the same order in which they are drawn."

DISCUSSION

Some Remarks on Haskell Fain's Paper "More on the esse is percipi principle". By *Konrad Marc-Wogau.* (Uppsala.)

In his article Mr. Fain criticizes my paper on Berkeley's well-known principle, published in this Journal in 1957. A few remarks on this criticism may be clarifying.

1. That the four interpretations of Berkeley's argument in *Pr.* 23 which I first consider are "four interpretations philosophers have offered" is not said in my text, and I do not know if they can be found in the Berkeley literature (possibly interpretation I). I discuss all five alternatives as possible interpretations, and I choose alternative V because this alternative seems most closely to agree with Berkeley's words.

2. In my text there occur a few formulas or expressions containing logical symbols. They are intended as summaries or "memory-aids". The context in which they appear should explain their meaning sufficiently. Taken by themselves, however, the formulas may seem objectionable from a pedantically logical point of view. Something should be said in answer to Mr. Fain's criticism of them.

a. The expression "exists" appears in these formulas as a predicate; Mr. Fain says that it should function as a quantifier. Although I do not myself regard "existence" as a predicate, I maintain that the point of many philosophical arguments is lost if one does not let "existence" stand for a predicate, but instead represents it by a quantifier: "there are" or "there is at least one x, such that". For instance, according to Leibniz' creation theory, there are monads which have existence. The expression "(Ex) (x is a monad)" cannot replace the expression "(Ex) (x is a monad and x has existence)". Something similar holds for Berkeley. According to him there are ideas and there are minds. The expression "there are" can be said to have the same meaning in both cases; for instance, the expression could mean that the class of ideas or of minds is not empty. But the "*esse*" or "*existence*" of an idea and the "*esse*" or "*existence*" of a mind have different connotations. And Berkeley's creation theory complicates matters still further. It is therefore advisable in the analysis of Berkeley's "*esse est percipi*"-principle to let "*esse*" stand for a predicate; this of course does not mean that "exists" cannot be represented by "(Ex) . . ." in many places in Ber-

keley and he himself replaces it by "there is" (cf. *Pr.* § 3). (The negation of the sentence "(x) (x *is perceived* $\supset x$ *exists*)" is clearly "(Ex) (x *is perceived* $\cdot x$ *does not exist*)"; this expression seems absurd only if one translates "Ex" by "There *exists* an x", without noticing that "exists" will then occur in two senses.)

b. Berkeley's sensationalistic thesis asserts, according to my text, that if something is perceived then this something exists with exactly the properties with which it is perceived. Mr. Fain's suggested interpretation of the thesis: "(y) (y *is perceived* $\supset$ (Ex) (x *is perceived*))" seems unsatisfactory without adding "$\cdot(x=y)$" after "x *is perceived*". Berkeley asserts that the negation of the thesis is "for me" a "plain contradiction"; this he does, in my opinion, because he understands the thesis as a consequence of the "*esse est percipi*"-principle which is valid according to him and not because he considers the thesis in itself as a logical truth.

c. I admit that my interpretation of premiss γ as a consequence of what I have called the extended sensationalistic thesis (1957, pp. 37 ff.) is a mistake. Certainly Berkeley thought of the possibility of making erroneous assumptions to the effect that something is the case. In fact he lets Hylas first mistakenly assume that there exists an idea which is not perceived. Assumptions such as these do not have the infallibility which belongs to sense perceptions. Since, however, in our discussion it is evidently a question of assuming something *truly*, premiss γ can still easily be explained. If I assume *truly* that a is unperceived, then it follows of course that a is unperceived.

d. Mr. Fain points out that the symbol "x" in the formulation of my interpretation V sometimes appears as a name, sometimes as a variable. It seems to me, however, that this inexactness need not lead to misunderstanding, if one keeps in mind what was said earlier about the structure of Berkeley's argument. "To prove the statement that every idea which exists is perceived, Berkeley endeavours to show the absurdity of supposing that there exists an unperceived idea. And in order to show this he discusses some examples. He argues as follows: it is absurd to suppose that the idea of a book exists unperceived, or that the idea of a tree exists unperceived, or that any idea whatsoever should exist unperceived" (1957, pp. 25 ff.). By going through individual sentences of the type "I suppose that x is unperceived" (where x is a name of an idea), all of which are seen to be contradictory, Berkeley wants to show that the sentence, "*I suppose that* (Ex) (x *is unperceived*)" (where "x" is a bound variable) is absurd. This structure of the argument, which is analogous to the one Mr. Fain himself thinks decisive, is not discussed in my interpretations II–V, although it is presupposed there. In the formulation of premiss γ "x" functions as a name.

3. The most important of Mr. Fain's objections seems to me to be his statement that Berkeley's argument is not intended as a demonstration. According to Mr. Fain, it is "a mistake to search for suppressed premisses or logical errors". But that Berkeley did not believe that he was demonstrating his principle, seems to be clearly inconsistent with his words in *PC*, entry 407 and in *Pr.* § 22 and *D.* II 200, where he explicitly talks about the demonstration of his principle. That his words in these contexts should be a remnant from an earlier point of view, where he believed that also the existence of spirits consists in their being perceived and where he thought he could demonstrate this belief, seems quite implausible. Evidence for such an interpretation must be considered as being very weak. The conception of the soul as passive in entries 577–581, to which Mr. Fain refers, does not correspond to anything in Berkeley's published writings, and can according to Luce, probably be explained as a mere terminological deviation from Berkeley's usual doctrine of the existence of spirits (A. A. Luce's edition of *Philosophical Commentaries*, pp. 426 ff.). (Moreover: is not Berkeley's demonstration, as Mr. Fain here interprets it, a *petitio principii*, since "*esse est percipi*" is deduced from the supposed synonymity of "there exists" and "there is perceived"?)

4. The interpretation of Berkeley's argument evidently depends on the relative weight given to his different statements. The assumption of "absolute existence of unthinking things" – – if such an expression can have any meaning at all – – is labeled by Berkeley as a "direct contradiction". "To talk of *conceiving* a thing which is *unconceived*" is also said, in *D.* (II 200), to be a contradiction and is there compared with "the contradiction to see a thing which is at the same time unseen". It is hardly reasonable to suppress the question wherein this direct contradiction consists.

REVIEW

Stephen C. Pepper: *The Sources of Value.* University of California Press, 1958. 699 p. By *Johan Hovstad †.*

The great merit of this new work by Professor Pepper is that it creates a synthesis out of the existing empirical value systems and thus makes it possible to utilize the valid propositions of several different value systems, which until now have been regarded as incompatible.

The new contribution to the value philosophy which makes this synthesis possible, is Professor Pepper's introduction of *several selective value systems,* and his construction of the conception of *survival values.*

Pepper belongs to the empirical school of value philosophy; he has much in common with Ralph Barton, Perry and owes much to his interest theory of value, but in his search for an answer to the question – what are the constituents of an interest – he comes to a solution which leads to a much more refined system of thought, where the concept of interest at last disappears, giving place to the representation of the activities of the dynamics of a purposive drive, and the bearing of these activities on the building up of what constitutes value.

As value empiricist Professor Pepper is related to the descendants of the ancient hedonistic school passing down from Hobbes through Bentham and Mill to Santayana and R. B. Perry. The originators of the conception of a general theory of value, the Austrians Meinong and Ehrenfels, also belong to this school. But through the introduction of the concepts of selective systems and survival value Pepper transcends the limits of the traditional hedonistic-utilitarian thinking, and thus seems to succeed in solving the most serious problems in ethics and general value theory, – the contradiction of the claims of individual and general happiness.

What then is a selective system? The main selection which human beings have to make in their daily life, is that between acts of appetition and acts of aversions. "The whole structure of an appetition is designed for the attainment of a terminal goal, and that of an aversion of the avoidance of a terminal object of riddance. These total structures are selective systems, natural norms, dividing an organism's environment, wherever they go into action, into terminal objects of positive value purposively sought after, and terminal objects of negative value

purposely avoided. This molar opposition of the essentially positive value function of an appetition and the essentially negative value function of an aversion is of ultimate significance. It is the first thing to catch the attention of the common sense observer, and it will be the last and most significant thing about purposive activity which we shall come back to in the end".

But a detailed study of purposive structures brings to light other selective activities within this total structure. The molar opposition of appetition to aversion that first strikes the eye, turns out to be in the nature of a resultant effect from the interaction of these other selective activities which operates inside the total purposive structure. So there are several selective systems.

These selective systems may differ from each other with respect to the dynamics or with respect to the chanelling through which the dynamics is structured, or with respect to both. Pepper's inquiry brings out that there are only two basic sources for the selective systems: one in the instinctive purposive drives, the other the vital force of evolutionary selection, the survival factor, which constitutes the survival value.

It is mainly through this factor that Pepper's synthesis, based on selective systems, seems to succeed in bridging the gap between egoistic and universalistic hedonism, which has been the unsolved problem of the traditional line of ethical thinking "the profound discrepancy between the natural end of a action, private happiness, and the end of duty, general happiness", to quote Henry Sidgwick, one of the most outstanding among the traditional hedonists. It was in despair over the possibility of bridging this gap that he gave up and resorted to intuitional ethics and a belief in the moral government of the world . . . in short Theism.

The introduction of the conception of the survival factor seems to bridge this gap. It has its ultimate source in the evolutionary principle of natural selection. Human survival in biological competition depends upon man's social solidarity. "As a solitary animal, man is remarkably weak and helpless. His survival, as cannot be too often noted in the study of values, depends upon his social relationships, which emerge in the form of cultural patterns. Cultural patterns are thus the principal biological instruments of survival for man. They are so effective that a great variety of cultural patterns serve this purpose, as we learn from the researches of antropologists. In fact, the most serious biological competitor with man is now man himself. And the cultural problem is not any longer to develop cultural patterns that will permit man to compete successfully with other biological species, but to permit man to compete successsfully with other men."

The essential thing for human survival is conformity to some cul-

tural pattern. Cultural conformity thus comes in with cultural patterns as a new selective element. Under emergency conditions the force for social security and human survival overrides the struggle for private happiness and utilizes the instinctive purposive dynamics which underlie this struggle toward the maximization of social security values. "This force for social security is attained by chanelling purposive drives through cultural patterns which institutionalize them, hardening them into social forces strong enough to break down egoistic personal drives that might rise up to challenge them.

What natural selection seems to have done for the human species is to have endowed it with this mechanism for institutionalizing personal drives into overindividual social forces strong enough to control these drives in the direction of social solidarity. An important part of this mechanism appears to be man's biological endowment with a capacity to communicate by means of syntactical language."

Here is not the place for a complete representation of Pepper's explanation of the development of these mecanics for the institutionalizing of our personal drives into overindividual forces. It suffices to mention that Pepper, by stressing that conscience is the incorporation into personality of the demands of the environing culture, maintains that it "has its roots in the repressions, and is consequently uneradicably involved in the irrational elements of the personality".

The survival factor is the pivotal agent of Pepper's theory of value. In a static and well established society the individual pursuit of happiness, the satisfaction of the purposive values which originate in our appetitions, naturally have the priority, but in emergencies, in times of danger and insecurity, especially in times of war, the survival factor will exercise its pressure on the individual, forcing him to sacrifice his own interests for the sake of the community and his fellowmen.

But also in peacefull times the dominance of the several selective systems can differ. Sometimes affective values – these are the values that in traditional language are associated with the conception of pleasure – are dominant, sometimes achievement value – the value of successful attainment of aspired objects – are dominant, and similarly with the selected values of the other selective systems. "It is the inconstancy of legislative dominance that has made the value field so controversial. Different ethical writers pitch upon different selective systems as the systems to be dominantly legislative over all the others. On occasion each of the principal selective systems can be found to operate in this controlling manner. Then confusion arises from the fact that champions for each systems can make about equally plausible claims. There are respectable schools of ethics pivoted on nearly every one of the principal selective systems."

Here the value of Pepper's selective systems becomes evident. The

dangerous nihilism of the cultural relativists is overcome, and thus their maintenance of the impossibility of finding objective ethical norms is refuted. Therefore, Pepper is very optimistic in his belief in the capability of value thinking to serve as a guidance in human affairs. Says he: "Values are thus as much open to description as any other objects in the natural world. These descriptions are verifiable so far as relevant data are at hand, and the search for further data is of the same order as the search for further data in any other empirical inquiry. In spite of the length of this book, all that the present study has done or could expect to do, is to direct attention to this empirical field of values, show where a quantity of relevant data are already to be found, suggest hypotheses for later confirmation, and solicit interest in seeking out more data and developing more refined hypotheses.

From this empirical study of values reliable predictions can be anticipated, and accordingly the possibility of a rational and humane guidance of man's social relations and personality development can also be anticipated."

But there is no danger of a rigid moralism as a consequence of this value system. The dominance of the different selective systems has to be adjusted to changing circumstances. Therefore, the ideal must be the adjustable society, which is a social invention of a new era. "It amounts to a new social species in the evolutionary history of man. It is the intelligent society, which bids fair to supplant all other forms of society from competition in its life zone, just as adjustable man through his purposive behaviour has eliminated all effective competition from other organic forms of life."

As an appropriate concluding passage of the review of this stimulating and enriching work Professor Pepper's description of the features that would distinguish the adjustable society, would serve very well: "Its social aim would be to maximize happiness, subject to the legislation of human survival values over affective values. It would seek to keep issues of survival and the pressure for survival as far away as possible. It would, therefore, under present world conditions, seek to organize all men into one society (without, however, destroying subcultural differences) in order to eliminate cut-throat competition among human societies. It would probably soon have to consider means of limiting populations, to preserve man from the social effects of population pressures, which are sure to invoke survival values and drive out affective values. It would do these things by regulatory social mechanisms administered by experts . . .

The inducement for men the world over to create such a society is strong. It is nothing less today than the survival of the human interbreeding populations."

Towards a theory of inference

by

JØRGEN JØRGENSEN
(University of Copenhagen)

In many textbooks of logic statements are spoken of in such ways that it seems reasonable to define: a statement is anything that can significantly be said to be true or false. In order not to waste time on subtle linguistic differences in the usage of the English word "statement" and the corresponding Scandinavian words "dom" and "omdöme" or the German word "Urteil" I take it for granted that the above-mentioned definition (whether expressly formulated or not) implies the following presuppositions: a) statements may be linguistically formulated sentences, b) statements are something that is either true or false, and c) one must know the meaning of the words "true" and "false" in order to know what a statement is. I shall start by making a few, by no means original, but important critical remarks concerning these presuppositions.

ad a) Statements need not be linguistically formulated. The linguistic formulation of a statement is *inessential*. One and the same statement may be expressed in different languages and even in different sentences within the same language. Besides, one may often be doubtful as to how to formulate a given statement in the most appropriate way, even when knowing very well what statement one wishes to formulate. All this indicates that the linguistic formulation is inessential to a statement, and that the essential thing is the *meaning* of the possible formulations or expressions, i.e. the apprehension or conception (often called the proposition or thought), which the apprehending or judging person has in his mind. One may, for instance, recognize a thing or a colour or a melody without formulating any designation of

them or any sentences describing them. On the other hand, many sentences, such as for instance interrogative, imperative and exclamatory sentences, are not expressions of statements. Thus, to state something and to use a language seem to be two processes that are in principle independent of each other, although they are very often connected in organisms that have an extensive training in using a language.

But what then is a statement?

To answer this question seems to be very difficult, and I shall therefore restrict myself to a very short description of what I consider the most essential characteristics of a statement. First, I think a statement is the result of a psychological process taking place in the person making the statement and nowhere else. Next, this process is directed towards an *object* in the wide sense of this word in which it may be said that a concept is a concept *of something,* a fear fear *of something,* hate hate *of something* etc. Further, the process makes one or more of the objects features appear, or appear more distinctly than before. The object towards which the process is directed is called *the subject of the statement,* and the features emphasied are called *the predicate(s) of the statement.* The statement does not contain the subject as a part, but it "has" the subject, i.e. it is directed towards the subject. In so far the process is a process of analysis: a given content of consciousness is articulated in such a way that certain features of it are emphasized as features or parts of the given totality. Which features are emphasized depend partly on the apprehending person and partly on the object apprehended. The judging individual cannot arbitrarily decide what features are to appear, i.e. to be found by the analysis. Although the individual might wish that the object should posses certain features he will be unable to create such features by analysis, since he can thereby only *ascertain* the existence of features that the object actually possesses. It may therefore, perhaps, be best to define: *to judge (or to make a statement) is to predicate, i.e. by means of a more or less thoroughoing analysis to ascertain that a given object contains or has certain features,* e.g. a certain quality, a certain magnitude, a certain position in space and

time, certain relations between its parts, etc. In so far the statement made may be said to be an answer to questions of the form: "What is this?", "What qualities has it got?", "When does it occur?", "What relations does it contain?", etc. But it is not necessary that such questions actually procede every act of judging, because the judging may occur so immediately that a state of uncertainty does not get time to develop itself at all.

If one wants to communicate one's statement to other persons, then one has to give expression to it by naming *which object* the statement concerns and *what is asserted* about that object. Often the most convenient way to do this is to use the usual linguistic expressions designating the object and its ascertained features. Since these designations are in principle different from the object and its features, they should never be called "subject" and "predicate" but always "subject-designation" and "predicate-designation". In the statement "the earth is round" the subject is the earth itself and the predicate a property of the earth, while the word "earth" is subject-designation and "round" predicate-designation. The word "earth" is not round, and the word "round" is not a property but a property-designation. This is, of course, obvious, but as we shall see later on, it is a thing which it is important to remark and bear in mind.

ad b) It is generally agreed that statements may be true or false. Nevertheless it seems incorrect to define statements as anything that may be true or false. Indeed, a general definition of a statement ought to state the characteristics that are to be found in *any* statement. But truth cannot be found in false statements, and falseness not in true statements: it is merely the alternative characteristic "true or false" that can be found in any statement. Just because these alternatives are different (and moreover are mutually exclusive) they merely give an "extensional" definition of statements, namely as an aggregate of two different classes, each of which is to be defined in its special way. Thus truth and falseness are *inessential* characteristics of statements (even if they play an important part in logic). Therefore I find it better to define: *a statement is an ascertainment of something (the subject) containing such and such features.*

Such statements may then be either true or false. But what does that mean? Under which conditions is a statement true and under which is it false?

ad c) Usually statement is said to be true, if and only if it agrees with reality. What then is reality? The word "reality" has, as is well known, various senses, the most important of which seem to be the following: In everyday-life it signifies *the usual reality,* and it is true to state that grass is green, the heaven blue, church bells noisy, roses fragrant, sugar sweet, etc. etc. But in science the word "reality" signifies *the objective (physical) reality,* and it is true to state that things are composed of colourless atoms, that light rays are electromagnetic waves, that sounds are air waves, that the measures of the spatial and temporal extension of the objects depend on the velocity of the observer in relation to the objects, etc. In psychology, however, the word "reality" signifies *the psychological or phenomenological reality* (what is immediately given), and it is true to state that the full moon is larger on the horizon than in zenith, that a thing's quality of being warm or cold varies with my temperature, that my images are colourless, etc. etc. Further, in *the world of literature, art, or imagination,* the "imagined" fantasies or daydreams are the "real" works of art or imagination, and it is true to say that Hamlet killed Polonius, that Ophelia went mad, that Pegasus is a winged horse, etc. etc. An finally, in *the sphere of mathematical objects* the word "reality" signifies the mathematical concepts, and it is true to say that in an Euclidean triangle the sum of the angles equals two right angles, that in a square the diagonal is incommensurable with the side, that continous functions that cannot be differentiated exist, etc.

If desiring to give the word "true" an unambiguous sense one thus cannot define the truth of a statement as its conformity or agreement with reality. But, if desiring to follow common usage, one cannot deny that all the above examples of statements are true, everyone of them expressing a correct or true apprehension of the object about which it predicates something, and its negation expressing an incorrect or false apprehension of the same object. The question then arises whether it is possible to find a

definition of "truth" that comprises all the various examples of true statements.

This question can, in my opinion, be answered in the affirmative. Common to all the statements mentioned is the fact, that the object about which they state something actually contains the predicate which they ascribe to it, i.e. that that predicate can be found by an analysis of the object: grass is something that is green, physical things are composed of atoms, the phenomenon of the full moon is larger on the horizon than in zenith, the imaginary animal Pegasus has wings, and the sum of angles in an Euclidean triangle equals two right angles, etc. In one word: in every true statement the predicate is contained in the subject of the statement, – no matter whether the subject is an everyday thing, a physical object, an immediately given content of an experience, an imaginary object, or a conceptual object. And, conversely, a statement cannot be true, if the object does not contain the features signified by the designation of the predicate. If the subject of a statement does not contain the features predicated of it, then the statement is false. If so, the result of a thoroughgoing analysis will show the falseness of the statement. Therefore, truth, respectively falseness, may be defined thus:

A statement is true, if and only if the predicate of the statement is contained in the subject of the statement; and a statement is false, if and only if the predicate of the statement is not contained in the subject of the statement.

If calling statements whose predicates are contained in their subjects "analytic statements" one may assert that *any analytic statement is a true statement, and that only analytic statements are true statements.* In this connection it must be emphasized that what we are here concerned with is *the subject* and *the predicate* of the statement, and neither with *the designation* of the subject or the predicate, nor with the judging individual's *concept* of the subject or the predicate. The fact that a predicate of a statement is contained in the subject of the same does not imply that the concept of the predicate is also contained in the concept of the subject. Whether this is the case or not depends

on how ample and correct a concept of the subject the individual has formed. Usually, however, the properties, relations or other features of the subjects of statements are not found by an analysis of the *concepts* of these subjects, but by an analysis of the subjects themselves, i.e. the objects about which something is predicated. Where, as, for instance, in mathematics, something is stated about objects that only "exist" by virtue of a definition, the subjects of the statements are concepts, the properties of which are to be found by a *conceptual* (logical) analysis of the definition (no matter whether this definition is formulated explicitly in a definitional equation, implicitly in a postulate-system, or merely understood).

According to Albertus Magnus the above-mentioned definition of the truth of statements was first proposed by the Arabian philosophers Avicenna, Alfarabi, and Alhazen ab. 1000 A.D. and was later taken up by Hobbes and Leibniz. As these thinkers did not, however, distinguish clearly between the subjects of statements and the concepts of these subjects, the statements were conceived as relations between the concept of the subject and the concept of the predicate, and the analysis was conceived as a purely conceptual one. In consequence, it lay near at hand for Kant later to introduce the presumably untenable distinction between what *he* called analytic and synthetic statements that play so important part in his whole philosophy. In my sense of the word there are no true synthetic statements. All true statements are analytic, but the analysis from which they spring varies in character as to *the sphere of objects* to which the subject of the statement belongs. If the subject belongs to *the sphere of everyday objects,* the analysis will be a usual observation; if it belongs to *the sphere of physical objects,* the analysis will be a more or less complicated investigation by means of various apparatuses; if it belongs to *the phenomenological sphere,* the analysis will consist in an awareness of what is immediately given; if it belongs to *the sphere of imagination,* the analysis consists in observing the characteristics of the imaginary beings or occurrences (by the way, one may here easily find examples of non-mathematical, undecidable statements, as for instance,

"Ophelia wore a pigtail"); and finally, if it belongs to *the conceptual sphere,* the anlysis will be of a logical character and often consist in a proof or disproof based on the given or presupposed definition.

The five spheres of objects here mentioned overlap, of course, and it may be very difficult to find precise criteria of each of them. But this is not essential for my present argument, no more than is the question as to whether they may be reduced to a smaller number or not. What matters is merely, that the decision of the question as to whether a given statement is true or not depends on some kind of analysis of the subject of the statement, no matter how many spheres of objects are concerned.

The next question that arises is: How many kinds of statements can be distinguished? This seems a very difficult question to answer, and I shall therefore restrict myself to mention a few examples that are of importance in this connection. First, it may be convenient, however, to distinguish between statements on the one hand and *designatory conventions,* that are *not statements,* on the other. Such conventions are, of course, necessary presuppositions for linguistic formulations of the statements, and, unfortunately, these conventions are themselves formulated in sentences that are identical with sentences expressing statements, e.g. "this is a tree", or "this is Mr. N.N.". Being *conventions* such sentences are, however, not expressions of ascertainments and can neither be true nor false. They may always be conceived as answers to the question: "What is this *called*?" or "What is the *name* of this?", and they should always be formulated by means of the words "called" or "named": "This is called a tree", or "N.N. is the name of this person" (if conceived as statements concering the usage of the language they are statements, but they are not statements concerning the designated objects, which remain unchanged independently of any arbitrary change of the designation).

Returning now to the real statements a most important distinction seems to be the one between *primary statements* and *secondary statements. Statements the subject of which is not statements, but other kind of objects, are called primary state-*

ments. And statements the subject of which is one or more primary statements are called secondary statements. Consequently, the statement "this is red" is a primary statement, and "the statement "this is red" is false" is a secondary statement. If necessary, this classification may be continued by the introduction of tertiary, quaternary, etc. statements.

Of other possible fundamental *fundamenta divisionis* may be mentioned:

a) the object-sphere to which the subject of the statement belongs;
b) the number of possible predicate-categories of the subject (about which something will be said further on), and
c) the extension of the subject.

These distinctions may be mutually crossed so that a rather complicated system of different kinds of statements result. Here, however, I shall confine myself to some remarks concerning the first-mentioned distinction, occasionally adding a little about the other ones.

The simplest and most fundamental of the primary statements are presumably those, the subject of which is a present object being analysed in such a way that either different features, or different parts of it and relations between such parts are ascertained. Statements in which the features are predicated of the subject may be called *attributive statements,* while statements in which relations between parts of the subject are predicated of the same may be called *relational statements.* Usually the linguistic expression of an attributive statement designates the subject by a single designation, while the several parts of the subject of a relational statement must have separate designations in the linguistic expression of such a statement. Both kinds of statements have, however, only *one* subject, i.e. the predicate(s) is (are) ascribed to one and only one subject irrespective of this having parts or not.

These two kinds of primary statements may also result from a process of recognition, in which a present object is ascertained to be the same as an earlier experienced object, or to be similar

to such object. Such *statements of recognition are necessary presuppositions for the introduction of designations of objects at all.* Without recognition of objects one cannot introduce and apply designations of them. But later on, when the application of the object-designations has been learnt, these designations often substitute the subject in linguistic formulations of statements, and these then appear to be purely linguistic affairs. The connection with the non-linguistic reality that alone gives the sentences a meaning is interrupted, and the designation of the subject or the concept of the subject may then easily be mistaken for the real subject, which mistake has caused serious errors in logical theory. The introduction of subject-designations is, of course, an enormous practical advantage, because by means of them one can talk about objects that are not present, – their names represent them. But as the names do not generally resemble the designated objects, one is, as a rule. forced to found one's predications on one's memories or on one's concepts of the objects concerned; and as one's memories or concepts are usually rather vague or meagre (and moreover often incorrect) one is, of course, easily led to rather unimportant or misleading statements. The name or the concept easily comes to function as the subject of the statement instead of merely representing it. Under such conditions the argument, at worst, develops into a purely verbal affair or, at best, into a purely conceptual construction (as is the case in all formal systems). Conversely, the subject-designation may be omitted from the linguistic formulation of the statement in cases where the subject is present to the conversing persons, or it may be referred to by a pronoun. Such statement-expressions without a subject-designation are in German called "Subjektlose Urteile" (subject-less statements), but this designation is misleading, since, as a matter of course, every statement has a subject – whether designated or not. A predication cannot be made without a subject. The sentence "it rains" is merely a linguistic shortening af another sentence that may be formulated somewhat as follows "my out-door environment is in a rainy state". On the other hand, many predicates of earlier formulated statements may be condensed in the subject-designa-

tion of a new statement, such as for instance, "the old man that lived in this house has died", which statement presupposes the truth of the statements "the man was old", and "the man lived in this house". And here it is immaterial whether the man belonged to the sphere of everyday things or to the sphere of imagination.

Returning to the classification of statements it is important to remark that primary statements (as well as statements of higher order) may be affirmed or negated. Affirmation of a primary statement is called *a positive (or affirmative) statement,* and negation of a primary statement is called *a negative statement.* Positive and negative statements are of the order next above the statements affirmed or negated. The statement " "this rose is red" is true" is a secondary positive statement, while the statement " "this rose is red" is false" is a secondary negative statement. Linguistically a negation statement may be shortened by means of the word "not" as, for instance, "this rose is not red". This shortened form of the linguistic expression has led to the untenable conception that negative statements ascribe a "negative" predicate to the subject. But "negative predicates" seem unthinkable, and thus all statements ascribe a positive predicate to some subject. If, however, the subject does not have the ascribed predicate, then the statement is false, and the negation of the same is then true.

A positive (secondary) statement is true, if and only if the predicate "true" can be found by analysis of its subject, which is a primary statement. The positive statement "the statement "this rose is red" is true" is itself true, if and only if the statement "this rose is red" is true, and this last-mentioned statement is true, if and only if the present rose actually has the quality red. The truth of the secondary statement is thus ascertained by an analysis of the corresponding primary statement, and the truth of this primary statement by an analysis of its subject. On the other hand, the property "true" can, of course, never be found by analysing merely the linguistic expressions of statements, since these linguistic expressions have but the property "true" in relation to their subjects, that can never occur as a part of the linguistic expressions (unless, exceptionally, the state-

ment is a statement about a linguistic expression, such as, "the word "statement" contains nine letters").

Analogously, a negative (secondary) statement is true, if and only if the predicate "false" can be found by an analysis of the corresponding primary statement. If so, the subject of the negative (secondary) statement (i.e. the primary statement) has the property falseness, and it is consequently true to ascribe this property to it. This is done in the negative statement, and therefore this statement is true.

True positive or negative statements presuppose true, resp. false, primary statements. And the fact that the truth or falseness of primary statements depends on the properties of their subjects, which properties are independent of the person making the statements, presumably show that truth and falseness cannot be introduced by convention, but are based on the ascertainment of facts. And analogously, where false positive or negative statements are concerned.

In so far as the truth or falseness of secondary statements depends on the truth or falseness of the corresponding primary statements, the first-mentioned may be said be *truth-functions* of the latter. Thus *truth-functions are a special kind of secondary statements*. Not all secondary statements are truth-functions. The statement "the statement p is complicated" is, for example, a secondary statement, but it is not a truth-function. In passing it may also be remarked, that not every statement containing another statement as part of it, is a truth-function. For example, the statement "A believes p" is not a truth-function, since it is not even a secondary statement. Its subject is not a statement, but the belief of A which is a psychological fact that may contain the statement p, but that is not itself a statement.

The relation between primary statements on the one hand, and positive, resp. negative, statements on the other may be expressed in a so-called truth-table:

If and only if the primary statement p is true, resp. false, then the positive statement "p is true" is true, resp. false, and the negative statement "p is false" is false, resp. true.

In this way we get a truth-table for affirmation corresponding

to the usual truth-table for negation, and it is shown that the truth-value of the truth-function eventually depends on the "content" (or "intension") of the primary statement, viz. on the properties of the subject, which properties are designated by the predicate-designation. If one does not take this "content" in account, then one does not know the sense of the words "true" and "false" at all, and the whole procedure changes into a meaningless play.

Affirmation and denial need not concern merely single statements, as was the case in the examples given above. Such statements may be combined either conjunctively or disjunctively, and by affirming or denying these combinations one gets new kinds of truth-functions, customarily called "conjunctions", resp. "disjunctions" ,that may be represented in the usual truth-tables, in which the arguments are primary statements and the functions secondary statements, viz. statements about the truth-values of the conjunctive or disjunctive combinations of the primary statements.

As regards the other usual truth-functions the matter seems to be more complicated. Take, for instance, the so-called *material implication* that is often defined as the disjunction of the negation of the first element on the one hand, and the second element on the other. As to this I may remark that it is, of course, possible to define such a truth-function and to call it "implication", but it is not so easy to make clear its connection with the processes of inference. If one defines "*p* implies *q*" as equivalent with (but presumably not having the same sense as) "either not-*p* or *q*", then it is clear that such definition will guarantee that *q* cannot be false, if *p* and "*p* implies *q*" are true. In so far "*p* implies *q*" is a *necessary* condition that *q* can be inferred from *p*. But it is certainly not a *sufficient* condition for such inference. What is wanted for a theory of inference is a relation that will allow one not merely to infer or deduce the truth of *q* from the truth of *p*, but even to deduce the "content" of *q* from the "content" of *p*. When making an inference from *p* to *q*, one does not merely infer the truth of *q* from the truth of *p*, but one first and foremost deduces the "content" of *q* from

the "content" of *p*, and it is actually the possibility of such a deduction of the "content" that guarantees that *q* is true, when *p* is true. It is possible to make an inference from a true statement to any other true statement, and sometimes it is possible to infer *q* from *p* even if *p* is false. This seems to show that *truth-values are not the important things in inferences. What matters is the "content" of the statements*. And as far as I can see, this "content" must be the *predicates* of the two statements. Between these predicates there may be relations that make it possible to find the "content" of *q* (i.e. the predicate of *q*) by analysing the "content" of the predicate of *p*, thereby simultaneously guaranteeing that *q* is true, if *p* is true. Thus, if the relation that makes an inference possible is called "entailment", then *entailment is an intensional relation and cannot be defined by means of truth-values* – neither by "truth" and "falseness" nor by "necessity", "possibility", "impossibility" or other modal terms.

How inferring is possible, is most easily seen by a consideration of the so-called *series of abstractions*, i.e. series of objects successively representing higher and higher abstractions from a given object. Examples are: square, rectangle, quadrangle, figure, something extended; physician, man, mammal, animal, organism, everyday-thing; work of art, beauty, value. Any object that suits one of these concepts (meaning of the words) will contain the features of which the following concepts are concepts. If, for instance, it is known that a given object is a physician, then it is possible to infer that that object is also a man, a mammal, an animal, an organism, and an everyday-thing. Such inferences are correct whether in the sphere of everyday objects physicians exist or not. The premiss "this is a physician" need not be true in order to make the inference to "this is a man" possible. But in order to be able to make such inference one must have formed such a *concept* of physician that this concept contains the concept "man" as a feature. If one has formed such a concept of physicians that it contains the assumption that a physician is able to cure any illness, then one is able to infer "this (man) can cure this illness" from the premiss "this (man) is a physician". The inference is correct and cannot be denied if its

presupposition is accepted, i.e. if one has formed such a concept of a physician as indicated, even if this concept is not in conformity with the facts. In this way it may be understood, how it is possible to draw conclusions from false premisses as well as from true ones, – a fact that agrees well with the fact that the variables in the logistic formulation of generalized statements are "apparent", i.e. that the truth-value of the statement is independent of those variables.

The facts here described seem to me to indicate the principle of all syllogistic inferences with general or singular conclusions. And the inference from the secondary statement "p is true" to the secondary statement "q is true" in the propositional calculus seems to run in a similar way. In such inferences the statement "if p is true, then q is true" is presupposed as the major premiss. But how can this premiss be proved? Obviously by means of an analysis showing that q is "contained" in p, i.e. that the predicate of q is a part of the predicate of p, and can thus be found by analysing this predicate. In the propositional calculus such analysis is, however, not carried through (and, being different for each single statement, it cannot, of course, be made generally, once for all, for all statements). One restricts oneself to *claiming* that the inference shall be made in agreement with the principle of *modus ponens*. But the application of this rule will, of course, not guarantee the truth of q, unless true statements are substituted for the figures ("signs") of p and *if p, then q*. In the formal sciences one confines oneself to *postulating* that such figures represent true statements without proving that this is in fact the case, and all consequences are therefore here also only postulates claiming that the deduced figures represent true statements. But whether the postulate mentioned is true or not can, of course, merely be known by trying if the figures can be interpreted in such a way that they become true statements concerning particular objects, and that means that their predicates can be found by analysis of their subjects. If such interpretation is not made, one does not know whether the figures represent true statements or not, indeed, one does not even know whether the theorems actually follow from the postulates (pri-

mitive propositions), because one does not know whether the figure *if p, then q* represents a true statement or not. A formal system of this kind is merely a play with figures that may be transformed in accordance with certain rules. But these rules are only entitled to be called rules of *inference,* if the figures are interpreted and the truth of the result of such interpretation is demonstrated.

In the inferences spoken of until now the major premiss was *either* an analytical statement *or* a pure figure, the representative force of which was merely a postulate. But the major premiss of such inferences may also be of another nature. A statement of the form "if p, then q" may namely be based on experience in the sense that in a number of cases it is ascertained (observed) that every time p is true, q is also true. If a recognized object that, in accordance with the conventions of significance, may be described by p, is repeatedly present, and if another recognized object that my be described by q, is present (or follows) every time, then these observations may be briefly described by "if p, then q". *This* will be in accordance with the truth-table for implication, and to this extent this truth-table merely summarizes the observations made of the connection of the two objects. The description "if p, then q" then is a *primary* relation statement, not a truth-function. The subject of this statement is the totality of the two objects (facts), that are described by the two statements p and q, respectively. The statement is not a statement about the two single statements or about some combination of the same, but it is a statement about the complicated totality containing as parts of it the two facts that are singly described by the statements p and q. The two objects may be, for instance, "rose" and "thorn", or "rain" and "wet street", so that we have "if rose, then thorn", resp. "if rain, then wet street". If such statements concerning the connection between two objects of nature are generalized, and if the generalization is true, it is called a "*law of nature*". The question as to how to justify such generalizations based on a limited number of observations is the problem of induction, of which I do not want to speak here. What I would emphasize is merely

that such generalization may be used as the major premiss in inferences from the truth of p to the truth of q. The principle on which such inferences are based is, that if the first term of an asymmetrical relation is present, then the second term of that relation will also be present or follow. If the statement "every time p is true, q is also true" is true, then the truth of p will guarantee the truth of q – even if it is impossible to find q by means of an analysis of p. This form of implication is thus an empirical and non-analytical ("synthetic") generalization of a summary of observations of the connection between the objects or facts described by p and q, respectively. However, the human mind seems to be of such a nature as to have a tendency (manifesting itself in everyday life as well as in the sciences) to make such generalized (so-called "formal") implications analytic by forming from the concepts of the subjects of p and q a new concept, from which the truth of q can be analysed. The concept of the subject of q is inserted into the concept of p, thus enlarging the content of the latter concept. Nevertheless, the former linguistic designation of the subject of p is often retained, but the definition of the same must, of course, be changed (by a so-called "successive definition"), if it is going to correspond to the enlarged concept. From the *designation* it is in such cases not possible to learn that the concept has been enlarged, but that this is so appears from the conclusions drawn from p. And it is always possible to express the enlargement linguistically in such a way that the inference gets a clear analytic obvious character such as, e.g., when the "synthetic" major premiss "if roses, then thorns" is changed into "thorned roses have thorns", or "if rain, then wet street" is changed into "the rainwet street is wet". Usually such obvious expressions are not formulated, but actually they express the major premiss in the obvious inferences from p (with enlarged concept of subject) to q. That the predicate of q can in such cases be found by an analysis of the enlarged concept is evident. But whether this concept can be applied to a given object and the conclusion thereby drawn shown to be true is, of course, a matter of experience.

From the above remarks concerning the processes of inference

it seems near at hand to regard *inferring as a special kind of judging* (making statements), i.e. as a kind of analysis of a given object. The difference between judging (in a narrower sense) and inferring will then be the one, that when a judgment (or statement) is being made, the subject of the judgment (or statement) is directly present, while in an inference the subject of a conclusion is merely represented by a concept, the process of inferring consisting in an analysis of this concept. The statement (the conclusion) resulting from this analysis must then predicate what can be found by such analysis of the concept of the subject, and nothing else, no matter whether this agrees well with the object about which something is inferred, or not. Conceptually we always conceive the objects through our conceptual spectacles, so to say. Even when *inferring* from a present object it is the *concept* of this object that is being analysed. Whether the result (the conclusion) is a true statement or not thus depends on the correctness of one's concept of the object. So, inferences are judgments that presuppose that the inferring individual has beforehand formed a certain concept of the object about which something is inferred, and directly the conclusion merely shows something about the inferring individual's concept of the object (his understood presuppositions), but not about the object itself, unless the concept agrees with the object. On the other hand, judgments in their narrower sense may be made without any preceding knowledge of the object. If I am seeing, e.g., a flower. I may either *judge* that it is red, fragrant, and a rose, or I may *infer* that it comes from a plant, that it has formerly been a bud, and that later it will wither, etc., – all depending upon how much I know about flowers beforehand, i.e. how ample my concept of flowers is. But all these last-mentioned qualities I may also *infer*, even if I do not see the flower but merely mobilize my concept of flowers generally, in so far this concept contains the qualities mentioned. If I have formed an incorrect concept of the object about which I am inferring something, my conclusions will probably be false. Even if they are correct or true statements about my *concept*, they will usually be false statements about their subjects. Unfortunately, I often fail

to distinguish between the objects that are the subjects of my statements, and my concepts of these objects, and I consequently draw false conclusions. I am usually drawing many conclusions about other persons, events, states, and other things, of which my concepts are insufficient or incorrect. But nevertheless I believe that my conclusions concerning these object are true, because I mistake my concepts for the objects, or suppose that they are correct, the main reason for such mistakes presumably being that I use the same *word* to express my concept and to designate its object.

Whether this conception of the nature of the kinds of inference hitherto spoken of is valid also as regards other kinds, can merely be shown by further investigations. I must here restrict myself to some suggestions.

As regards inferences from two relational statements we may consider the following example: "if A is larger than B, and B is larger than C, then A is larger than C". We have here two premisses with a common term, i.e. a totality must be present, and in this totality three terms or parts must be distinguishable, two of them having a certain relation to the third. Then a relation between the two extreme terms can be found by analysing the totality. Also in such cases we must make a distinction between the relational complex itself and our concept of it. Presumably the origin of our relational concepts is experience (observations) of relational complexes. And when drawing a conclusion from relational premisses, the resulting relational statement can be found by analysis of the relational complex present. But once the relational concepts have been formed, the "new" relation can be found solely by analysis of the *concept* of the totality described by the premisses. What relational concepts we form depend on our experiences. They may be largely different as e.g. *logical relations* (difference, entailment, similarity, part-whole, etc.), *spatial relations* (above-below, right-left, before-behind, short-long, etc.), *temporal relations* (before-after, simultaneity, short-long, etc.), *relations of kinship* (mother-child, father-child, sister-sister, sister-brother, grandparents-grandchildren, uncle-nephew, cousin-cousin, etc.), or *relations of magnitude*

(larger-smaller, equal in size, etc.). All of them, however, seem to allow inferences conceived as processes of analysis.

About inferences in which class-concepts occur a few special remarks may be made. First, a distinction must be made between *intensional classes* and *extensional classes*. Intensional classes are collections or manyfolds of objects having one or more "common" qualities, i.e. being of such a nature that it is possible to find these common qualities by an analysis of *each* of the objects (the objects are "similar" in one or more respects). Such classes are e.g. those that are usually linguistically designed by the common nouns ("men", "animals", "stars", "squares", etc.), whereas the linguistic collective nouns designate extensional classes (to be spoken of in a moment). Of every member of an intensional class it is true to predicate the qualities that are common to the members. Often such predication is expressed by means of a so-called *general affirmatice sentence* of the form "all S are P", the meaning of which is that *every* member of the class S has the quality P (not that the *class* S has this quality, – probably no class can have any quality that can be predicated of its members). – In the logic (calculus) of intensional classes it is the relations of class inclusion that matter: if a class A is included in a class B, and if the class B is included in the class C, then the class A is included in the class C. The conclusion here means that every member of the class A has the qualities that are common to every member of the class C. The subject of this statement thus is "any object having the qualities common to the members of A", and the predicate is "the qualities common to the members of the class C". The conclusion is true if and only if the predicate can be found by analysis of the subject, and that this is actually the case becomes obvious when the premisses are analysed analogously. The general conception of inferences as processes of analysis thus seems to hold also with regard to inferences of this kind, and the same may be said of inferences in which one of the premisses is a *general negative* or a *particular (positive or negative) statement*.

Let us now consider inferences containing extensional classes. Such classes may perhaps be best defined as collections or ac-

cumulations of objects that need not have any quality in common. The membership of the class may be indicated by merely pointing at the objects or by uttering the names of the same. If the objects have not got a name, one may use the words for the natural numbers as names. By means of these words it is possible to number or count the members of extensional classes and to define the relation of magnitude between them: the later in the series of numbers the position of the last number used in the process of numbering is, the larger is the class numbered. These relations of class-magnitudes may be described in relational statements allowing inferences: "If A is larger than B (i.e. if A contains more members than B), and if B is larger than C, than A is larger than C". The three classes may, however, have members that are quite different in kind, wherefore such inferences do not tell anything about the question whether the class C is contained in the class A or not. The membership of C may be quite different from the membership of A, and A nevertheless be larger than C. In fundamental sections of mathematics (the theory of manyfolds, or sets, and arithmetics) extensional as well as intensional classes play an important role and are often connected in very complicated ways.

In conclusion, I would make i few remarks concerning what I call *categories of predicates*.

Consider a simple everyday thing, e.g. a table, and let the analysis of same result in the finding of the following properties: rectangular, four-legged, brown, glossy, hard, wooden, combustible, large, etc.

All these properties are compatible, i.e. they may be found simultaneously in one and the same thing, and the result of the analysis may be expressed in a single true sentence: "This table is rectangular, four-legged, brown, glossy, hard, wooden, combustible, large, etc.".

If now we compare this table with other tables, we may find that they have partly the same properties, partly different properties. One of the other tables may, for instance, be round, three-legged, white, glossy, harder than the first one, of marble,

incombustible, small, etc. A third table may be triangular, one-legged, grey, dull, hard, of stone, medium-sized, etc.

All the properties found by analysis of the various tables may now be classified in different groups which I shall call *categories of predicates,* because each property corresponds to the predicate in one of the statements that may be made concerning one or another of the tables. The principle according to which the classification is carried out is, that predicates that are mutually exclusive belong to one category. *A category of predicates thus comprises the predicates that exclude one another in statements concerning the same subject.* No subject has more than one predicate from any category. Experience shows which predicates are incompatible.

In the above-mentioned statements concerning the tables, predicates from the following categories are contained:

the category of form: rectangular, round, triangular;
the category of colour: brown, white, grey;
the category of leg-number: four-legged, three-legged, one-legged;
the category of reflection: glossy, dull;
the category of hardness: hard, still harder;
the category of material: wood, marble, stone;
the category of combustibility: combustible, incombustible;
the category of size: large, small, medium.

If we do not confine ourselves to the analysis of tables, but include all possible objects, we shall find a lot of qualities belonging to various categories of which there is an immense number: the category of weight, of duration, of stability, of sound, of taste, of various kinds of value, etc. etc.

Of these categories of predicates the following general statements seem to be true:

Every subject (object) may have predicates belonging to various categories, but there hardly exists any object having predicates from *all* categories. Quadratic numbers, for example, cannot be red hot, tables not hard-hearted, atoms not green, etc. Predicates belonging to a category the members of which cannot

be ascribed to a certain subject may be said to be *foreign to this subject*. And statements ascribing such foreign predicates to a subject may be said to be *meaningless*, in contradistinction to *false*, false statements ascribing a predicate to a subject to which it is not foreign, but merely actually not belonging. Of a quite *concrete* subject it is always meaningless to predicate properties that it actually does not possess (a concrete white swan cannot be both white and black). But of a more or less *abstract* subject it is always meaningful to predicate the various properties belonging to a category of predicates of which a member can be ascribed to the corresponding concrete subject (swans, generally, may be either white or black).

In passing it may be here remarked that also questions may be meaningless, in case they presuppose that a given subject may have a predicate foreign to it. Answers to such questions are neither true nor false, but meaningless statements in the above-mentioned sense. The elaboration of the logic of questions is, as far as I know, a task that has not yet been accomplished.

Predicates belonging to the same category may be disjuncted, but not conjuncted, i.e. of the same subject it will always be true to assert that it has either the first, or the second, or the third, etc. predicate of one of its possible categories, every category comprising also the negation of the other predicates *within* the category, but this negation *not* being identical with any predicates *outside* the category. For instance, "shapeless" is neither identical with "colourless", nor with "immaterial", nor with "thoughtless", nor with any other predicate belonging to another category than the one to which the predicate "shape" belongs. Every category form so to say a universe of predicates, and it is such a predicate-universe that is split up when represented as the logical sum of a term in it and the negation of that term (this negation being the disjunction of all the other terms in the universe). Thus *the universe of truth-values*, for instance, is exhausted by the predicates "true" and "false", because "false" is the negation of "true" and *vice versa*. Every statement the predicate of which is the disjunction of all possible predicates within a certain category is a tautology in Wittgenstein's sense.

Such statement must always be true, because one of the predicates must belong to the subject. If this was not the case, all the predicates would be foreign to the subject, and the statement consequently meaningless. So is the nature of our conceptual universe and of our corresponding significations of predicates. The fact that the predications must be restricted to a definite category, Wittgenstein, however, seems not to have observed. But it can hardly be denied that it is highly artificial to predicate of a given subject a disjunction of predicates from *different* categories, as e.g. "the table is either rectangular, or brown, or four-legged, etc.". Indeed, these predicates do not exclude one another, and in common usage they are not disjuncted. Therefore, the non-exclusive disjunction, introduced by Jevons for calculatory reasons, must be considered an artificial device deviating from the logic of everyday language. Corresponding to the above-mentioned statement concerning the disjunction of predicates it may thus be stated:

Predicates belonging to different (but possible, i.e. not subject-foreign) categories may be conjuncted, but not disjuncted. While it is contrary to usage to say "the table is either rectangular, or brown, or four-legged, etc.", it is quite in order to say "the table is rectangular, and brown, and four-legged, etc.". This last-mentioned statement would, however, be contradictory, if the negation of a predicate was not restricted to the corresponding category of predicates, i.e. if "four-legged", for instance, belonged to the negation of "rectangular". Whether the last-mentioned conjunctive statement is true or false must be decided by an analysis of the subject concerned. Statements, however, in which a conjunction of predicates belonging to the same category is predicated of a subject, are always false and thus have a certain affinity to contradictions in Wittgenstein's sense.

Actually, the characterization here given of tautologies and contradictions seem to show that the present concepts of same are generalizations of the corresponding Wittgensteinian concepts. According to Wittgenstein tautologies and contradictions are respectively disjunctions and conjunctions of *statements*, and what matters are the *truth-values* of these statements, while

here we have to do with disjunctions and conjunctions of *predicates generally* of which the truth-values are special instances. The statement "it rains or it does not rain" is a special instance of the general scheme "either *p* is a true statement, or *p* is a false statement", this being a tautology because the predicates "true" and "false" exhaust *the category of truth-values* of the statement. Of course, statements may have predicates belonging to other categories; they may, for instance, be more or less complicated, be primary or secondary, or tertiary, etc., be attributive or relational, etc. But in the statement "any statement must be either true or false", merely the category of truth-value is considered. As this category contains merely the two predicates "true" and "false", it may properly be said to be divalent, and its two members to be *contradictory*. Other categories of predicates may be *trivalent, tetravalent,* or generally *polyvalent,* and in all these cases the members of the categories are said to be *contrary*. Contrary predicates exclude one another, but merely taken all together they exhaust the category. An inference from the falsity of a statement to the truth of its negation therefore becomes more and more indefinite, the more comprehensive the category of the statement's predicate is. If the category is divalent it is possible to infer from the absence of one of its members to the presence of the other member, e.g. from the absence of the predicate "false" to the presence of the predicate "true", and *vice versa (tertium non datur)*. But if the category is trivalent, then the absence of one of its members will merely allow an inference to the presence of the one or the other of the two remaining members without telling which of them can be ascribed to the subject of the statement. And in case the number of predicates in the category increases, the indefiniteness also increases, this fact becoming particularly conspicuous where gradually changing qualities are concerned, such as, hard and soft, large and small, heavy and light, etc.

As to polyvalent propositional logics, they are, in my opinion, *either* purely formal (uninterpreted) games, *or* dealing with predicates not belonging to the category true-false. If a third truth-value, e.g. "indefinite", is introduced, the two other values should

presumably not be named "true" or "false", but rather "definitely true" and "definitely false", and the subject of these predicates will then be the judging person's knowledge of the truth-values of the statements, not these statements themselves. Similarly, in case a propositional logical formalism with infinitely many truth-values is interpreted as a logic of probability, the extreme members of the category of probability are not "true" and "false", but "true with the probability 1", and "true with the probability 0", and between these two extreme values all the other probability-values are situated. That the statement p has the probability m/n $(m \leq n)$ means that the predicate of p may be ascribed to n subjects, but that merely m of these subjects have this predicate, i.e. that p is true merely in m of the n possible cases. Or more exactly: when I ascribe the probability-predicate m/n to a statement p, then I am merely predicating of p that it belongs to a group of n statements of which m and not more than m are true. If $m=n$, then p has the probability 1, and it is possible to infer that p is true, because it must then belong to the group of true statements. But nevertheless the concept "true" and the concept "true with the probability 1" are quite different, and the last-mentioned of these concepts can merely be defined by means of the first-mentioned. And analogously as regards the concepts "false" and "false with the probability 0".

Finally, I may add that the various categories of predicates mentioned in this paper are but a small selection of a vast number of such categories which have mutually numerous and complicated relations. The above-mentioned series of abstractions are but a single, and exceptionally simple, class of such relations. As far as I know, this vast field has until now been but little investigated. Presumably the linguistic semanticists have done more in this direction than the logicians. And possibly such investigations can throw some light on the Hegelian-Marxist "dialectics", which are actually a continuation of some of Aristotle's ideas that have been widely neglected by later logicians. If my modest suggestions in this paper could stimulate the interest in the problems here touched on, they would not have been written in vain.

Abus alcoolique et délinquance

Étude fondée essentiellement sur des expériences suédoises

par

OLOF KINBERG
(Stockholm)

Il est connu depuis des siècles que l'excès d'alcool peut influencer les comportements humains de façon désastreuse. Le mouvement de tempérance qui a été depuis longtemps très actif, surtout dans les pays anglosaxons et scandinaves, a été suscité par cette expérience et a fait prendre des mesures législatives prohibitives aux Etats-Unis et dans quelques pays nordiques (Finlande, Norvège) dont les conséquences sociales et morales se sont révélées très néfastes (consommation illégale excessive de boissons alcooliques, nouvelle occasion de gains presqu'illimités suivie d'une criminalité très dangereuse se révélant entre autre par les luttes acharnées qui mettaient aux prises des organisations concurrentes dans les grandes cités américaines, mépris des lois prohibitives qui s'étendait à d'autres secteurs de la législation, etc.).

Comme le montre un aperçu de la littérature qui se rapporte aux liens entre l'alcoolisme et la délinquance les études statistiques de ce problème manquent de bases solides.

Certes, un grand nombre de données sur la fréquence de l'alcoolisme dans différentes catégories de prisonniers ont été publiées par différents pays. Elles varient pourtant beaucoup non seulement de pays à pays, mais aussi d'auteur à auteur dans un même pays. En outre, les critères sur lesquels se base le diagnostic de l'alcoolisme sont peu connus.

En Suède où la lutte contre l'alcoolisme date de loin, une commission officielle désignée en 1944 pour étudier la question a tâché d'établir le caractère des habitudes de consommation en

étudiant un échantillonage répresentatif de 135.000 sujets. La commission a obtenu des renseignements tirés du casier judiciaire de 41.646 hommes ayant au moins 25 ans.

Ce groupe a été réparti en six catégories selon les habitudes des sujets à l'égard des boissons alcooliques.

Le premier groupe englobait ceux qui avaient fait l'objet de mesures antialcooliques prises par les conseils antialcooliques communaux au cours des trois dernières années. 24–48 % de ceux-ci avaient des peines; les chiffres obtenus variaient selon les différents types de communes où ils avaient vécu.

Le deuxième groupe comprenait d'autres cas d'alcoolisme grave où des sujets qui avaient subi au moins trois peines pour ivresse sur la voie publique pendant les dix dernières années; 28–34 % de ceux-ci avaient été condamnés pour d'autres genres de délits.

Le troisième groupe était composé de personnes qui avaient été punies pour ivresse deux fois au plus en dix ans, ou qui avaient commis d'autres contraventions alcooliques pendant les dix dernières années. 13–19 % de ceux-ci se retrouvaient dans le casier judiciaire.

Dans le quatrième groupe, formé par des personnes qui consommaient avec modération des boissons alcooliques, la proportion de condamnés était de 3–6 %.

Dans le cinquième groupe, composé de sujets presque abstinents, 3–6 % avaient été frappés de peines.

Dans le sixième groupe, celui des abstinents, la proportion correspondante était de 1–3 %.

Selon la Commission, le risque de commettre un délit d'une telle gravité qu'il entraîne l'insertion dans le casier judiciaire est pour les trois premiers groupes respectivement 60,47 et 19 fois plus grand que pour les groupes des abstinents.

Pourtant, eu égard au fait qu'aucun délit n'est causé par un seul facteur, on se tromperait en soutenant que la grande difference relevée entre ces catégories et alléguée par la commission ne dépend que du facteur alcool. Il est certain que l'attitude des différents sujets en ce qui concerne la consommation des boissons alcooliques dépend de leur structure personelle, c'est-a-dire des

facteurs biologiques dont est composé leur alliage constitutionnel et des modifications que differentes lésions cérébrales ont peut-être apportées à celui-ci.

Il est bien connu que plus le noyau constitutionnel est composé harmonieusement, à savoir, plus un sujet possède les traits structurels nécessaires à une maturation mentale, mieux il est adapté aux conditions de la vie. Cela implique non seulement que ses pulsions délictuelles sont faibles, mais aussi que sa fonction morale est bien développée de sorte qu'il est rebuté par les habitudes qui offensent la dignité humaine, entre autres par celles qui consistent à abuser des boissons alcooliques. Donc, les habitudes de sobriété et l'absence de délinquence son très souvent entraînées par les mêmes facteurs. Le connexe que suggère l'énoncé de la commission est donc mal fondé.

Le problème des rapports entre l'alcoolisme et la délinquance, en dépit des nombreuses publications qui le traitent, reste encore assez mal élucidé. Cela nous a incité à entreprendre avec nos collaborateurs G. Inghe et T. Lindberg une étude un peu plus étendue.[1] Elle a été réalisée à l'Institut criminologique de l'Université de Stockholm, avec le soutien généreux de l'Association des brasseurs Suédois.

Les éléments qui composent la matière de cette étude sont des personnes qui ont fait l'objet de mesures anti-criminelles au cours de 1947, réparties comme suit:

Les individus qui ont été conditionnellement condamnés avec liberté surveillée; les sujets condamnés à des peines privatives de liberté; les internés des établissements de sûreté; les sujets condamnés à être traités dans des établissements affectés aux jeunes délinquants (traitement correspondant au système Borstal); les sujets déclarés exempts de peines à cause de troubles mentaux.

La matière traitée englobe 6.612 hommes et 503 femmes, soit un total de 7,115 cas.

Les informations relatives aux abus alcooliques proviennent de renseignements relevés à l'occasion de différents actes où se

[1] O. *Kinberg, G. Inghe, T. Lindberg,* Kriminalitet och alkoholmissbruk (Abus alcoolique et criminalité). Stockholm 1958.

sont manifestées des habitudes de consommation alcoolique [ou d'abstinence]; intoxication à l'occasion du délit; condamnations à cause d'ivresse sur la voie publique; délits commis en état d'intoxication alcoolique (3,6 % ayant été punis pour ivresse au volant, 8,6 % pour d'autres délits); cas traités dans des établissements pour alcooliques (5,6 %).

En considérant les catégories délictuelles on a pu constater que l'intoxication à l'occasion du délit était le plus répandu parmi les délinquants qui avaient commis le crime de viol. Dans les autres catégories, il est apparu que les malfaiteurs violents montraient la plus grande fréquence non seulement d'ivresse sur la voie publique mais aussi d'intoxication au moment du délit. Les délinquants ayant commis certains crimes contre la propriété (faussaires, ceux qui avaient détourné des valeurs) avaient le plus bas pourcentage d'alcoolisme. Parmi les voleurs, par contre on a trouvé un pourcentage moyen. Le nombre des alcooliques chroniques qui avaient commis des attentats aux moeurs était peu élevé tandis que celui de ceux qui avaient été intoxiques au moment du délit était assez élevé.

Dans les groupes sociaux où la délinquance est peu répandue on a remarqué que l'alcoolisme chronique et l'intoxication à l'occasion du délit étaient assez fréquents parmi ceux qui avaient délinqué, ce qui montre que dans cette catégorie il faut une augmentation de la pulsion où une réduction de la résistance, ou l'une et l'autre pour qu'un délit se produise. Aussi est-il connu que la fréquence de troubles mentaux est assez grande chez les délinquants appartenant à une couche sociale où la délinquance est plutôt rare (Inghe 1941).

Ces observations sont importantes du point de vue criminogène. D'abord, on trouve un connexe marqué entre l'abus alcoolique et la délinquance parmi les jeunes. Chez les délinquants âgés, le même phenomène se retrouve; mais eu égard au fait que les habitudes alcooliques sont plus répandues dans cette dernière catégorie l'abus est donc plus répandu parmi les jeunes criminels. Cela s'explique par le fait que ces deux formes de comportement se conditionnent mutuellement: d'un côté les sujets alcooliques se trouvent souvent dans des situations précriminelles

plus ou moins spécifiques, où leur résistance contre les pulsions est affaiblie; de l'autre, les délinquants deviennent facilement des alcooliques.[2] Evidemment, les deux formes de comportement peuvent dépendre de facteurs individuels entraînant des symptômes de désadaptation différents qui favorisent une évolution simultanée vers la délinquance et vers l'alcoolisme. Tout cela aggrave le prognostic social: la jeunesse désadaptée retombe facilement dans des habitudes alcooliques; les délinquants alcoolisés récidivent facilement. De cela l'on peut conclure que le traitement de cette catégorie de délinquants doit viser l'un et l'autre des symptômes de désadaptation: la délinquance et l'alcoolisme.

Notre étude a montré aussi que l'alcoolisme chronique joue un rôle beaucoup plus grand dans la catégorie des récidivistes que dans celle des délinquants débutants. La déchéance sociale et morale engendrée par des lésions du cerveau et autres lésions alcooliques est un des facteurs prépondérants du récidivisme. Les récidivistes apartiennent à un déchet social où le travail honnête et les sentiments de responsabilité morale envers autrui n'existent pas. L'indifférance qu'ils éprouvent à l'égard du lendemain et des évaluations morales prévalentes dans la société, leur manque d'ambition et d'appétitions normales et leur besoin permanent de se procurer des boissons alcooliques amènent des délits répétés. Ils sont habitués à la vie de prison qui, par ce fait a peu d'effet déterrant. A cause de tout cela leur délinquance est le plus souvent futile: de petits larcins, de petites escroqueries, recel, etc. Pourtant, dans bien des cas ou la structure personnelle possède certains traits prédisposant à des réactions violentes, on trouve souvent des crimes de violence: rébellion et outrage, coups et blessures parfois suivis de la mort de la victime.

Notre étude a révélé un autre fait intéressant, à savoir que l'abus alcoolique et l'intoxication à l'occasion du délit sont assez rares dans certaines catégories de délits. Ainsi chez les prévaricateurs on trouve rarement l'alcoolisme chronique ou l'intoxication à l'occasion du délit, ce qui n'est pas étonnant. Les sujets

[2] Cf. O. *Kinberg*, Les problèmes criminologiques fondamentaux. Paris 1958.

qui peuvent commettre des délits de prévarication forment un groupe assez restreint composé d'individus qui mènent pour la plupart une vie sobre et qui se trouvent dans des situations précriminelles qui ne permettent pas d'intoxications aigües.

Il est plus remarquable que le nombre des alcooliques soit assez restreint parmi les délinquants sexuels. Bien des délits de ce genre (inceste, exhibitionisme, etc.) sont d'ailleurs souvent commis par des sujets qui sont assez bien adaptés, ce qui a été révélé aussi par d'autres études, entre autres par notre monographie sur les incestueux.[3]

D'autre part, on trouve souvent lorsqu'il s'agit de ce type de délits des lésions cérébrales qui jouent un rôle prépondérant. Surtout chez les individus qui ont commis des crimes graves de ce genre, par exemple l'inceste entre père et fille mineure, le père est presque toujours un grand lésé du cerveau. Autre fait intéressant; ces criminels sont souvent en état d'intoxication aigüe, surtout la première fois qu'ils commettent le crime. Pourtant, les incestueux que nous avons étudiés sont souvent des sujets travailleurs qui appartiennent à un groupe dont le niveau social est peu élevé (valets de ferme, ouvriers dans les petits métiers, etc.). Le fait que l'intoxication aigüe est fréquente chez ces criminels montre que ces délits présupposent un affaiblissement de la résistance, peut-être aussi un renforcement de la pulsion par une stimulation de l'instinct sexuel. Il se peut aussi que l'indifférence à la souffrance de la victime soit nécessaire à la genèse de beaucoup de ces crimes.

*

Cette étude porte sur les délinquants d'une année où l'abus alcoolique, mesuré par rapport au nombre de tous les sujets condamnés pour ivresse sur la voie publique équivalait à la moyenne des années 1921–1940, environ 30.000. Ce nombre peut être considéré comme une expression de l'effet obtenu par les mesures législatives introduites sur l'initiative d'un médecin suédois, le docteur Ivan Bratt. Le but de cette législation peut se résumer

[3] *O. Kinberg, G. Inghe, Sv. Riemer,* Incestproblemet i Sverige (Le problème de l'inceste en Suède). Stockholm 1943.

ainsi. La vente de boissons alcooliques à emporter ne serait permise qu'à des sujets dont on savait qu'ils n'étaient pas des alcooliques ou qui ne montraient pas de tendance à l'alcoolisme. Pour qu'il fût possible de passer au crible ces sujets, il fallait faire une enquête sociale préalable sur chacun de ceux qui demandaient l'autorisation de faire de tels achats. Le droit de vendre serait réservé à une seule société soumise à un contrôle sévère de la part de l'Etat. La quantité de boissons alcooliques distillées qu'un individu pouvait acheter dans ces boutiques contrôlées par l'Etat était limitée à trois litres maximum par mois.

La vente de boissons alcooliques dans les restaurants ou autres débits serait limitée, de sorte qu'aucun client n'aurait le droit d'acheter plus de 15 centilitres de boissons distillées au cours d'une visite au débit.

Enfin, la vente des boissons alcooliques dans les restaurants et débits serait organisée de manière à ce que les vendeurs n'eussent aucun intérêt économique à inciter les clients à une consommation accrue.

Cette législation fut adoptée successivement à partir du 1er. janvier 1914.

Les effets ne se firent pas attendre. Le chiffre des condamnations pour ivresse sur la voie publique, qui était de 59.000 en 1913, baissa progressivement et finit par se stabiliser, en 1921, à 30.000 environ; il n'y eut plus ensuite que des variations insignifiantes jusqu'en 1942.

Cette législation avait deux catégories d'ennemis:

1) Les sujets alcoolisés ou ceux qui avaient un goût pour l'alcool assez fort pour voir d'un mauvais oeil toute limitation de la vente des boissons alcooliques, et en outre ceux qui considéraient la législation comme un empiètement intolérable sur la liberté individuelle.

2) Les grands groupes d'abstinents hostiles à tout compromis avec l'alcool, qui possédaient une presse active et exerçaient un pouvoir politique considérable.

Sous l'influence d'une campagne de longue durée et menée avec la force du fanatisme, ces deux facteurs réussirent à miner

la législation restrictive du système Bratt de sorte qu'au printemps 1954 le Riksdag l'abolit et rendit pratiquement libre la vente des boissons alcooliques. La nouvelle loi entra en vigueur le 1er octobre 1955.

Cette décision du Riksdag eut des conséquences désastreuses. Les condamnations pour ivresse, qui se montaient à 41.000 en 1954 passèrent à 79.000 en 1956. Cela signifie que le nombre des condamnations qui donnait un pourcentage de 1045 sur 100.000 en 1913 avait atteint celui de 1072 sur 100.000 en 1956, c'est-a-dire *le maximum de condamnations qui ait jamais été enregistré en Suède*. En 15 mois on avait donc perdu ce qu'on avait gagné par 43 années d'application d'une législation restrictive à laquelle la majorité de la population s'était habituée et dont elle comprenait les effets salutaires.

D'autres conséquences de la libération totale de l'alcool ne manquèrent de se manifester. La morbidité alcoolique s'accrût rapidement à côté de l'accroissement de la consommation des buveurs, de sorte que les maladies alcooliques connurent une extension effrayante.

Il est probable qu'à présent 2 à 3 % de la population masculine est composé d'alcooliques. Le nombre d'alcooliques reçus dans les asiles, qui était de 741 pendant les premiers six mois de 1954 monta à 1670 pendant la période correspondante de 1956. Durant ces espaces de temps les nombres de personnes souffrant de délire alcoolique étaient 55, resp. 332. L'augmentation du nombre des alcooliques placés dans des asiles déjà surchargés rendit ceux-ci encore moins capables qu'auparavant de recevoir les malades mentaux pour lesquels ils sont organisés.

Un autre aspect funeste de cette liberation de l'alcool est l'augmentation de l'abus alcoolique parmi les jeunes. Ainsi la proportion de condamnations pour ivresse sur la voie publique qui était de 2,6 sur 1000 sujets entre 15 et 17 ans pendant 1953 est monté à 5,6 pendant 1956.

Les établissements affectés au traitement des alcooliques sont devenus tout à fait insuffisants. Les Conseils antialcooliques communaux sont surchargés d'alcooliques dont ils ne peuvent pas s'occuper.

Les buveurs que sous le régime du système Bratt étaient exclus de l'achats d'alcool dans les boutiques contrôlées par l'Etat et qui trouvaient leur *quantum satis* dans les restaurants ont découvert à présent qu'ils peuvent boire à meilleurs marché en achetant leurs boissons dans les magasins de vente. C'est pourquoi ils prefèrent boire à domicile, si bien que les beuveries ont lieu dans les familles plus souvent qu'auparavant avec les conséquences désastreuses que cela comporte pour les épouses et les enfants des buveurs.

Des faits résumés ci-dessus on peut conclure: que les alcooliques invétérés boivent encore plus qu'auparavant; que leur nombre a augmenté; que l'accès plus facile aux boissons alcooliques a favorisé ce développement chez les sujets disposés à devenir des buveurs.

Quelle que soit la forme adoptée par un pays pour la vente des boissons alcooliques, personne n'ignore que ce ne sont pas tous les habitants d'un pays qui deviennent des buveurs. Il existe donc un choix dont les causes biologiques sont encore pour la plupart ignorées. Il est pourtant évident que ces facteurs relèvent de la structure personnelle des sujets. Les adhérents de l'hypothèse selon laquelle une législation prohibitive serait le seul moyen de combattre l'alcoolisme préconisent donc une foi dénuée de fondement.

Cette foi simpliste les amène à suréstimer la signification des chiffres de la consommation totale d'un pays, calculés sur la quantité consommée par individu et année. A notre avis, une telle méthode peut fort bien renseigner sur le dommage économique produit par la vente de l'alcool, mais elle ne dit presque rien sur le dommage qu'elle porte à la santé du peuple, à sa morale, à la vie de famille, à l'éducation, à la sécurité des habitants contre des attaques criminelles contre les personnes ou les propriétés, etc.

Pour mesurer les dommages individuels produits par l'abus alcoolique il est selon nous nécessaire de recourrir à une notion bien différente, celle de la *densité de consommation*, c'est à dire la quantité d'alcool consommée pendant l'unité de temps. Cet indice varie énormement d'un genre de consommateur à l'autre.

En Suède il existe un type de consommateur qu'on pourrait nommer «les chevaliers du goulot collé aux lèvres», c'est à dire des hommes qui mettent la bouteille à la bouche et boivent jusqu'au moment où ils sont arrivés à une ivresse profonde ou à la perte de la connaissance, et ceux qui ayant commencé à boire ne peuvent s'arrêter que lorsqu'ils se trouvent dans un état où il leur est impossible de continuer. Il existe donc dans chaque population un certain nombre de sujets qui par leur structure personnelle sont prédisposés, ou presque prédestinés à devenir plus ou moins rapidement de grands buveurs.

Sans nul doute, l'omniprésence de l'alcool favorise un tel développement. On lâche ainsi les brides à la foule de tendances antisociales ou nettement criminelles qui sont endiguées chez les gens sobres par des mécanismes inhibiteurs. C'est précisement ce cercle vicieux qui a été mis en fonction par la libération funeste de l'alcool.

En faisant l'expérience dangereuse de libérer les boissons alcooliques fortes, le Riksdag a jeté du bois sur le bûcher de la criminalité; il a même soufflé sur le feu. Presqu'au même moment, la Faculté de Médecine de Paris demandait au gouvernement français de prendre les mesures les plus énergiques pour combattre l'alcoolisme.

An interpretation of logical formulas

by

JEAN A. PHILLIPS

(Sherman Oaks, Calif.)

---

1. *Introduction.* The interpretation which we shall propose is unconventional in two respects. 1) Logical expressions (with the exception of those containing free variables) may be interpreted in either of two ways: a) When occurring as parts of larger expressions, they are interpreted as class names. b) An expression as a whole, when used as a sentence, is interpreted as the assertion that the class named in a) is the universal class. 2) Propositional connectives are interpreted in terms of class relationships involving the class of non-universal classes and the classes named by the expression flanking the connectives.

In spite of these two differences, our interpretation does not result in a new reading of formulas; it serves rather as an analysis of an old one. This interpretation will be dealt with in detail in Sec. 2.

In Sec. 3 it will be shown that, with this class interpretation of propositional connectives, propositional logic is not merely formally isomorphic to class logic; it is itself a class logic. This class logic is not restricted to two values. For truth and falsehood are not taken as discrete values: they correspond, rather, to the two divisions, universality and non-universality, of a range of values.

In Sec. 4 the implications of our interpretation for a theory of denotation will be considered. We will show how a sentence may be constructed as a name of a fact (or class of facts), where a fact consists of a relationship or pattern in a class. A sentence is true if and only if this class is universal. On this view a false sentence denotes the same kind of fact as a true one, the only

difference being that the realm to which it applies is not the whole universe; a false sentence thus denotes an incomplete fact.

2. *The Interpretation of Formulas.* In the following formulas, we shall use the letters at the beginning of the alphabet as constants, those at the end as variables. The constants will be interpreted as names denoting classes. The variables will be interpreted, following Quine [6: 70] not as denoting variable classes, but merely as indicating cross reference. Formulas with unbound variables, then, do not denote anything and are meaningful only by virtue of functioning as parts of larger formulas. Formulas can be interpreted either as class names or sentences. We shall start with the class interpretation of formulas containing no variables.

The formulas $\bar{a}$, $a \cup b$, and $a \cap b$ are interpreted as names of the complement, union, and intersection of classes. The formula $a \subset b$ is the name of the class of entities which if in a are in b. (This class contains all entities which are not in a; it is also named by the formula $\bar{a} \cup b$.) With this interpretation the formula $a \subset (b \subset a)$, ordinarily treated as unmeaningful, can now be interpreted as the name of the class of entities which if in a are members of the class of entities which if in b are in a. The formula $a = b$ is the name of the class of entities which are members of both a and b or of neither of them.

The formula $\sim a$ is the name of the class containing all non-universal classes plus the complement of the unit class of a. The symbol $\sim$, preceding an expression, has the effect of converting it from the name of a universal class into the name of a non-universal class, and vice versa. For if class a is universal, class $\sim a$ omits the universal class a and is thus non-universal; if class a is non-universal, class $\sim a$, which contains all non-universal classes plus everything but a non-universal class, is thus universal.

The formula $a \cdot b$ may be interpreted as denoting the same class as $a \cap b$. The formula $a \downarrow b$ may be defined as $\sim a \cdot \sim b$, and thus names the class containing all non-universal classes and all classes except the unit class of a and the unit class of b. The

formula $a \supset b$ may be defined as $\sim(a \cdot \sim b)$ and thus names the class consisting of all non-universal classes plus the complement of the unit class of the common part of 1) class a and 2) the class containing all non-universal classes plus the complement of the unit class of b. The formula $a \equiv b$ names the common part of this class and the class named by the formula $b \supset a$. The formula $a \vee b$ names the same class as $\sim(\sim a \cdot \sim b)$.

The formula $a \in b$ names the class containing all members of b and the complement of the unit class of a.

An expression in which the symbol ( ) occurs may also be interpreted as the name of a class: The formula (x) ... names the class of things x such that ... is the universal class. Thus the expression (x)(x$\in$a) names the class such that the complement of the unit class of each member in turn plus the class a is the universal class. Since this class is universal only for x's which are members of a, it is simply the class a. Note that this interpretation of the formula does not require that the symbol x be assigned a denotation; it is only the context in which it occurs which is significant.[1] The quantified variables need not occur in the remainder of the expression. The formula (x) ($a \subset b$) names either the universal class or the empty class, depending upon whether or not class $a \subset b$ is universal.

Expressions with any number of variables may be interpreted in this manner, as long as all occurences of variables are bound. Consider the expression (x)(y)(x loves y). For a particular choice of x, the part (y)(x loves y) would name a class – the

---

[1] There is an alternative interpretation which allows quantified symbols to have denotation rather than being mere expository devices: Treat the letters at the end of the alphabet, as well as those at the beginning, as constants naming classes. The formula (x) ... can then be interpreted as naming the class of entities which satisfy the condition imposed upon x by the requirement that the remainder of the expression name the universal class. Here, although the symbol x denotes, its denotation is not relevant to the denotation of the expression as a whole; the prefixing of the expression (x) cancels out the occurences of x in the remainder of the expression. In a system thus interpreted no variables are needed.

This interpretation was suggested by Fitch's treatment of abstraction [2: 97 ff.].

class of all things loved by x. (x)(y)(x loves y) then names the class containing those choices of x for which (y)(x loves y) names the universal class, or in other words, it names the class of things which love everything. The expression (y)(x)(x loves y) names the class of things which are loved by everything. Note that in reversing the order of the quantifiers we have changed the denotation of the expression. However, if either expression names the universal class, the other does also.

The symbol ˆ can be interpreted the same as ( ) when it occurs singly. However, double abstraction is not defined as two successive operations and thus must not be identified with double quantification;[2] the expression $\hat{x}\hat{y}$ . . . names the class of ordered pairs x and y such that . . . is the univesral class.

Other symbols of the logic of analyzed sentences can be defined in terms of the above symbols and thus can be given class interpretations.

The above interpretations apply to all expressions which occur as parts of formulas. Any expression as a whole may also be interpreted as the assertion that its class is universal. When this class is indeed universal, the sentence is true; otherwise it is false. Sentences asserting classes to be universal are ordinarily reformulated so that the word universal does not appear.

The formula $\bar{a}$ asserts that the complement of class a is universal, or in other words that class a is empty. The formula $a \subset b$ asserts that the class of things which if in a are also in b is universal, or in other words that all a are b. The formula $a = b$ asserts that the class of entities which are both a and b or neither is universal, or that a and b have the same membership.[3] The formula $(a \subset b) \cap (c \subset d)$ asserts that the common part of 1) the class of entities which if in a are in b and 2) the class of entities which if in c are in d, is universal, or in other words that all a

[2] In a system in which ( ) and ˆ are defined formally as identical, the notation $\hat{x}\hat{y}$ could be replaced by $\widehat{x;y}$ to avoid confusion with (x)(y).

[3] Up to this point, the relationship between class names and sentences indicated here has already been suggested by Hilbert and Ackermann [3: 46 f.]. They do not, however, develop this topic in connection with formulas containing other symbols.

are b and all c are d. The formula $(a \subset b) \subset (c \subset d)$ asserts that class $a \subset b$ is included in class $c \subset d$; this relation is sometimes expressed: If all a are b, then all c are d.

When one is not concerned with the internal structure of the parts $a \subset b$ and $c \subset d$, the conjunction and conditional just displayed can be symbolized simply as $d \cap f$ and $d \subset f$, where the symbol d names the same class that was named in the formulas above by the expression $a \subset b$. In the formulas of propositional logic, individual symbols are interpreted as names of complex classes.

The formula $\sim a$ asserts that the class of non-universal classes plus the complement of the unit class of a is universal, or in other words that class a is not universal. The formula $\sim (a \subset b)$ can be reworded as: a is not included in b.

To assert $a \supset b$ is to assert that the common part of 1) class a and 2) the class containing all non-universal classes and the complement of the unit class of b, is non-universal. This may be reworded as the assertion that if 1) is universal, then 2) is non-universal. Since the non-universality of 2) depends upon the universality of class b, this assertion may again be reworded: if a is universal then b is universal. The formula $(a \subset b) \supset (c \subset d)$ asserts that if class $a \subset b$ is universal, then class $c \subset d$ is universal, or in other words, if a is included in b, then c is included in d. This is a weaker assertion than the conditional symbolized above in terms of class inclusion.[4] Our basic interpretations of $a \vee b$ and $a \equiv b$ as assertions can also be reworded into their familiar renditions.

The assertion that class $a \in b$ is universal amounts to the assertion that class a is among the members of class b. To assert that class $(x)\ (x \in a \supset x \in b)$ is universal is to assert that all things are, if members of a, also members of b. Similarly, other sen-

[4] In the case of a conditional analyzable as a class inclusion, if the consequent is false (i.e., if its class lacks certain elements), then the antecedent must be false for the same reason (i.e., its class must lack the same elements) in order that the statement as a whole be true. However, in this weaker kind of conditional, it is sufficient when the consequent is false, that the antecedent also be false, regardless of which elements are lacking. Thus there need be no connection whatever between the facts described by the antecedent and consequent.

tences treated in conventional systems can be translated into a form in which a class is asserted to be universal.[5]

The interpretations given above would be modified when applied to a particular formal system, depending upon what restrictions are used to avoid paradoxes. Applied to a system in which the symbols are classified according to rank, it would be necessary to differentiate expressions accordingly; in particular, the expressions "universal class" and "class of non-universal classes" would refer to one of a heirarchy. Also, the dot of conjunction would need to be differentiated from the class product, as the former would be used to link expressions of different rank. The application of our interpretation to Quine's system as presented in [5] would, for example, require such modification.[6]

[5] Quine, in [6] analyzes simple sentences into statements asserting class relationships; it has been shown above that assertions involving propositional connectives can also be translated into statements about classes. A class statement may in turn be analyzed as an assertion of the universality of a class.

[6] A formula in which two expressions of unequal rank are joined by a dot could be used as the name of the common part of two classes of the same type if the dot were interpreted as raising the type of the class named by the expression of lower rank. For example, the conjunct a'.b could be used as the name of the common part of 1) class a' and 2) the class of non-universal classes of objects plus the unit class of b. The conjunct a".b would name the common part of 1) class a" and 2) the class of non-universal classes of classes of objects plus the unit class of: the class of non-universal classes of objects plus the unit class of b. Since 2) is universal (non-universal) whenever b is, the conjunct names a universal class if and only if both a and b are universal.

With this change, our interpretation may be applied to Quine's system in [5]. For his nine definitions still serve the purpose for which they are used and his five primitive statements are still true. The foundations would need to be altered as indicated below, but this would not affect the workings of the system.

Quine [p. 146] describes his definitions as introducing conventions of notational abbreviation. Since D1, D3, and D4, using our interpretation, link expressions which are equivalent (i.e., they name classes which are both universal or both non-universal) but not identical in denotation, we would label these conventions rules of substitution. The expressions linked by these rules appear in contexts where only the universality or non-universality of the class named (and not its specific denotation) is relevant

Since our interpretation can be applied to his system, we shall not present a formal system here. However, if we were to develop a system to serve as a natural vehicle for this interpretation it would have class notions basic. All logical symbols can be defined in terms of the two primitives $\uparrow$ (the class counterpart of Sheffer's stroke) and $\hat{}$ (or ( )), in a way which conforms to our interpretation.[7] What primitive statements and what precautions against paradoxes would best be taken in a system with these two undefined symbols is beyond the scope of this paper.

---

to the truth of the whole statement; hence substitution in true statements will result in true statements.

That these expressions are equivalent may be illustrated by applying our interpretation to an instance of his first definition: $a''' \supset b'$ for $\hat{x}\ a''' \subset \hat{y}\ b'$. The expression $\hat{x}\ a''' \subset \hat{y}\ b'$ is a class inclusion between two expressions, where each names either the universal or empty class of objects. When a names a non-universal class or b names a universal class, this inclusion will name the universal class; otherwise it names the empty class (and thus a non-universal class). The expression $a''' \supset b'$ names the class of non-universal classes of the 3rd type plus [the complement of the unit class of the common part of class a''' and {the class of non-universal classes of the 2nd type plus the unit class of (the class of non-universal classes of the 1st type plus the complement of the unit class of b')}]. This is also a universal class if and only if a names a non-universal class or b names a universal class. Thus these two expressions are equivalent.

Quine's definition of expressions used as name matrices link expressions which are identical in denotation. Thus the substitution of one for the other in contexts in which the membership of classes as well as their universality is relevant is warranted.

[7] The five class symbols $-$, $\cup$, $\cap$, $\subset$, and $=$ can be defined in terms of $\uparrow$ in the way in which their propositional analogues are defined in terms of $\downarrow$. (See Quine [6: 47 f.].) ιa can be defined as $\hat{x}\ (x=a)$, V as $\hat{x}\ (x=x)$, (x) as $\hat{x}$ (see footnote 2), $\exists x$ as $\sim(x)\sim$, $a \in b$ as $\iota a \subset b$, $\sim a$ as $a \in \overline{\iota V}$, $a.b$ as $a \cap b$, $a \supset b$ as $\sim(a.\sim b)$, and so on.

If expressions are to be classified according to rank, the symbol $\uparrow$ would be used to join expressions of the same rank only. The dot would then function to equalize the ranks of the expressions it joins. To translate into unabbreviated form, the rank of one member of the conjunction would be raised to the rank of the other and the dot replaced by $\cap$. For example, the expression $a'''.\ b'$ could be considered the abbreviation of $a''' \cap \{V'' \in \iota(V' \in \iota b')\}$.

3. *The Nature of Propositional Logic.* The reasoning of propositional logic has the same formal characteristics as class logic.[8] Applying the interpretation given in Sec. 2, it also has

[8] Rosenbloom [9: 30] concurs here, supporting his position by the fact that ". . . the logic of classes and the logic of propositions are models of the same deductive system, namely that of Boolean algebra". However, there are theorems of propositional logic (e.g., $a \supset b . \vee . a \supset \sim b$) which have analogues (in this case, $a \subset b$ or $a \subset \bar{b}$) which are not theorems of Boolean algebra. To establish the formal isometry between class and propositional reasoning, we must compare propositional logic to a system involving class reasoning only.

Hilbert and Ackermann [3: 46] take a system whose interpretation as a propositional logic is familiar. Then they reinterpret this system in its entirely as a class logic. The resulting pure class logic contains many formulas which are unmeaningful using the usual class interpretation (formulas with hierarchies of connectives) but which are there interpretation as we have interpreted class expressions in Sec. 2. This class system obviously has the same formal structure as propositional logic.

Incidentally, this class logic is so limited that it must be used in conjunction with propositional logic in order to handle the reasoning of Boolean algebra. Hilbert and Ackermann [3: 46 f.] discuss the possibility of using one system for both kinds of reasoning, where variables are interpreted as being replaceable by either class names or sentences and where the two interpretations are related in that the sentence asserts that the class consists of all objects. Wajsberg [11: 116 ff.] gives the foundations for such a system. However, his treatment has certain shortcomings – e.g., his system has formulas which are true but do not have meaning.

We have considered an alternate way of obtaining a system combining both kinds of reasoning. Start with a pure class system such as is obtainable from Hilbert and Ackermann by replacing all propositional symbols by class symbols. Instead of giving these symbols a second interpretation, extend the system to cover propositional logic, taking into acount the relationship in meaning between class and propositional connectives as interpreted in Sec. 2. This could be done by adding the primitive $\sim$ and defining the other propositional connectives in terms of $\sim$ and the class connectives: $a \supset b$ as $\sim a \subset \overline{\sim b}$, etc. Then add a rule which will allow the inference of one expression from another whenever the relationship in meaning between $^-$ and $\sim$ warrants it. (When $\bar{a}$ is universal, a is empty; and $\sim a$, which denotes all non-universal classes plus everything but the empty class a, is universal. The rule must thus allow the inference of $\sim a$ from a, also $\bar{a} \subset b$ from $\sim a \supset b$, etc.) Such a rule of inference unfortunately turns out to be quite complex, and the derivation of propositional logic out of class logic requires more space than available here.

the same subject matter as class logic – relationships between classes in the same universe. The difference between the two systems is that the propositional logic relates classes to the class of non-universal classes while the class logic does not. The fact that we use only the categories truth and falsehood in classifying sentences does not imply that propositional logic is limited in some way in which class logic is not. For truth and falsehood account for a whole range of values – a sentence is true or false depending upon whether its class is universal or non-universal.[9] The propositional logic is thus two-valued in only a trivial sense, just as every person is one of two ages, either over 21 or not.

The view that propositional logic is analogous to a class logic whose variables are restricted to the values 1 and 0 is taken by Lewis [4: 78–89]. Lewis shows how the propositional logic may be derived as an analogue of a class logic (the Boole-Schröder algebra) consisting of class variables and connectives, the constants 1 and 0, and non-symbolized words. He obtains the propositional logic by replacing the class variables by propositional variables and the class connectives and non-symbolized words by propositional connectives, except in the case of certain equal signs which are used to express identities. Lewis then comments [4: 79] "... there is one further principle, which does not hold for classes ... which is required in order to restrict the terms of the algebra to statements which are propositions: namely, 'For every element a, either $a=0$ or $a=1$'. If we add this assumption, and its consequences, to the Boole-Schröder Algebra, we have what is called the Two-valued Algebra." He then adds the postulate $(p=0)\equiv\sim p$ and its consequence $p\equiv(p=1)$.

This postulate does indeed restrict the values which p can assume. However, had he replaced *all* class symbols by propositional symbols, the weaker assumption $(p\equiv 0)\equiv\sim p$ would have sufficed to obtain the propositional logic. This formula does

---

[9] Since universality and non-universality exhaust all possibilities, there is no gap between truth and falsity to be filled. Thus an extension of the system to allow the expressions to take on values other than truth and falsity is not indicated.

not restrict the propositional logic to the two values 1 and 0.[10] For the expression $p \equiv 0$ does not assert that p is empty, but merely that p and 0 are equivalent. By our definition of equivalence two classes are equivalent if they are both universal or both non-universal. Since the empty class is non-universal, to assert that $p \equiv 0$ is to assert only that p is non-universal. The formula $\sim p \equiv (p \equiv 0)$ asserts that the expressions flanking the major connective name classes which are both universal or both non-universal. The formula is self evident, given our interpretation. Its only function is to permit the elimination of constants from the system. Thus propositional logic can be derived by analogy from class logic without imposing restrictions which limit it to two values.

Quine [8] shows how a two-valued propositional logic may be derived out of a class logic by defining propositional connectives in terms of class connectives and abstracts. For example, the implication $p \supset q$ is defined [p. 45] as a class inclusion between $\hat{x}\, p$ and $\hat{y}\, q$ (where x does not appear in p, nor y in q). He then identifies the truth value of the sentence p with the abstract $\hat{x}\, p$, remarking [p. 121] that "The truth value of a proposition is *the universal class or the null class* according as the proposition is true or false". He concludes [p. 127] that "...

[10] Rosenbloom [9: 50], in deriving a Boolean propositional logic, defines the constants rather than taking one as primitive. The statement $(p \equiv 0) \equiv \sim p$ then follows as a theorem, and the need for Lewis's special assumption vanishes. Rosenbloom remarks [p. 49], in connection with the view that in propositional logic there are only the two distinct propositions, 0 and 1, "This is manifestly wrong since no special assumption as to the number of elements of C is forced upon us".

In supporting the possibility of other values besides 0 and 1 he discusses other systems: non-categorical logics (with their undecidable statements) and modal logics (with their distinctions other than those expressible in terms of truth values). However, he does not discuss the significance of this possibility in connection with propositional logic, which is categorical and which deals only with the relation between truth-values of propositions.

We give significance to this possibility by not identifying falsity with 0, holding rather that a sentence is false whenever its class has a value different from 1.

the propositional calculus *is* the class calculus in confinement to the universal and null classes; the agreement is not merely one of structure, but one of subject matter as well".

Quine has shown that a system with the structure of propositional logic can be derived out of class logic. However, his method of derivation imposes an unorthodox interpretation upon the system. By identifying truth-values with values of abstracts, Quine equates his interpretation with the usual interpretation and draws the conclusion that since the expressions of his system can take on only two values, that propositional logic is therefore two-valued.

We find it questionable whether the identification between truth-values and values of abstracts conforms to what is ordinarily meant by truth. These abstracts were probably introduced not to throw light upon the nature of truth, but merely to facilitate the formal derivation of a system. If we reject this identification, Quine's interpretation must be treated as a new model of propositional logic rather than a paraphrase of the usual interpretation, and no conclusions may be drawn from it as to the number of values which apply with the usual interpretation.

Our view of truth (namely, that the truth value of a sentence depends upon whether the class bearing the relationship described by the sentence is universal) is also unorthodox; however, it follows naturally from our treatment of meaning. With this view of truth, propositional logic may take on the same range of values as an unlimited class logic.

4. *A Theory of Denotation.* We hold the view that a sentence describes or denotes a fact. This view is a natural one in the case of simple positive true sentences. However, other kinds of sentences such as denials and disjunctions are not so easily accounted for by such a simple theory, for we do not find negative facts or alternativity in nature. False sentences also raise a problem, for a sentence is commonly regarded as false when there is no fact to support it.

One solution to the first difficulty is to treat denials, etc., as statements about linguistic facts,[11] in which case the sentence

[11] Russell [10: 79] equates "not-p" with " 'p' is false".

"Not all swans are white." would be analyzed not as a statement about swans, but as about the sentence "All swans are white." On our theory, denials denote facts in the non-lingustic world, facts which differ from those denoted by simple positive sentences only in their complexity.

The problem of the denotation of false sentences could be handled by linking them to non-factual entities of one sort or another.[12] We prefer, however, to treat false sentences not as accurate statements about some ideal or psychological world, but as inaccurate statements about the factual world. For us a false sentence denotes an incomplete fact; its denotation differs from that of true sentences only in its scope.

In the discussion which follows, we assume a class analysis of sentences in which they are translated so as to contain only names denoting classes, variables, quantifiers, propositional and class connectives, and other symbols which are definable in terms of these.[13] An expression built out of these symbols can, as shown in Sec. 2, be used as the name of a class. Used as a sentence, it attributes universality to this class. Now to attribute universality to a class whose parts are in a certain relation amounts to asserting of the universe that its parts are in this relation. We suggest that a sentence describes or denotes this relationship, which we shall call a fact.

For example, "whales$\subset$mammals", used as a class name, is

[12] Quine asks [7: 472 f.] "But what manner of things are these, whose names are sentences? Not fact, for that would leave no place for false propositions. Are they then judgments? Or abstract possibilities, Platonic ideas? Or are they merely, as with Frege, the two truth-values, truth and falsity?" He then dismisses the problem by saying "... the whole notion of sentences as names is superfluous and figures only as a source of illusory problems. Again [6: 32], "A statement remains meaningful, but meaningful by virtue of its structure together with the meanings of the constituent names and other words; its meaningfulness does not consist in its being a name of something." Others, however, concern themselves with the meaning of a sentence as a whole. For Carnap, a proposition is an objective, non-mental, extralinguistic entity which may or may not be exemplified in nature [1: 27 f.]. Russell, finding no objective entities to link false sentences to, suggests that propositions are entities in a subjective realm [10: 229, 237 f.].

[13] Quine assumes such an analysis in [6].

rendered "entities which if whales are mammals", and thus denotes a class whose subclass consisting of whales is included in its subclass consisting of mammals. Used as a sentence, it asserts that this relationship between whales and mammals holds universally. The sentence denotes this relationship.

When a sentence is true, its class is the universe; when false, its class is less than the universe, and thus the pattern in the class does not pervade the universe. The sentence still denotes an actual pattern, but a pattern to be found only in the class of the sentence. Thus a false sentence might be said to denote an incomplete fact. For example, the sentence "Swans$\subset$white." denotes the fact of inclusion between swans and white things in so far as it exists – i.e., it denotes this pattern in the class containing everything but non-white swans.[14]

An assertion of the form a=b describes or denotes the relationship of equality holding in the class of things which are both a and b or neither. An assertion of the form a$\in$b denotes the relationship of class inclusion between the unit class of a and class b, or in other words, the relationship of membership between a and b. When this assertion is false, the class of the expression excludes a, and the relation holds vacuously (for the null class is included in any class).

The denial $\sim$a denotes the relationship of membership between class a and the class of non-universal classes. To take a simple example, the sentence "Not all swans are white", which is analyzed as "$\sim$(Swans$\subset$white), denotes the fact of membership between a) the class containing both non-swans and white things and b) the class of non-universal classes. A truth value conditional of the form a$\supset$b can be translated into $\sim$(a$\cdot\sim$b). The expression a$\cdot\sim$b denotes the common part of class a and the class containing all non-universal classes and the complement of the unit class of b. The assertion $\sim$(a$\cdot\sim$b) then denotes the relationship of membership between class a$\cdot\sim$b and the class of non-universal classes.

[14] That the author of such a sentence believes the sentence to apply to the whole universe is indeed part of the total meaning of the sentence, but we are here concerned with only one aspect of meaning, namely, denotation.

An expression containing a quantifier denotes a class of facts. The expression (x) ... asserts that the class x such that ... is the universal class is itself the universal class, or in other words that all things satisfy the relationship described by .... This expression then denotes a class of relationships or facts, where each fact pervades the universe and each fact involves a different entity x. If the assertion is true, the class contains a fact for each element x of the universe; if false the class omits certain of these facts.

Consider the expression (x)(y)(x R y). For a particular x, the part (y)(x R y) asserts that x bears the relationship R to everything, and thus denotes the class of facts describable by such sentences as: x_1 R y_1, x_1 R y_2, x_1 R y_3, etc. The whole expression asserts that every x satisfies this class of relationships. It thus denotes the class of facts describable by such sentences as: x_1 R y_1, x_1 R y_2, x_1 R y_3, etc.; x_2 R y_1, x_2 R y_2, etc.; x_3 R y_1, etc.; etc. In the class of facts denoted by the whole expression, each fact pervades the universe, and each class of facts involving a particular x includes a fact for every element y in the universe. If the assertion is true, there will be a class of such facts for every element x in the universe.

This theory of denotation has several implications for philosophy. 1). All true sentences have the same subject matter – the universal class; however, they denote different relationships in this class. 2). The denotation of false sentences is located in the same realm as that of true sentences, the difference being that the relationships denoted hold in classes which are less than universal. Thus error is accounted for in the world of fact. 3). A denial is about the factual world if the corresponding positive statement is about the factual world. 4). The denotation and truth value of a sentence are distinguished, the denotation serving as a criterion for the truth value. 5). The denotation of a sentence depends upon the denotation of its parts, and the structure of language, when analyzed as suggested above, corresponds to the structure of fact.[15]

[15] This statement is reminiscent of Wittgenstein [12]. However, our positions differ widely in detail.

BIBLIOGRAPHY

1. Carnap, Rudolf, *Meaning and Necessity,* Univ. of Chic. Press, 1947.
2. Fitch, Frederic Brenton, *Symbolic Logic,* Ronald Press Co., New York, 1952.
3. Hilbert, D. and Ackermann, W., *Principles of Mathematical Logic,* Chelsea Publishing Co., New York, 1950.
4. Lewis, C. I. and Langford, C. H., *Symbolic Logic,* Century Co., New York, 1932.
5. Quine, W. V. "Logic Based on Inclusion and Abstraction", *J.* of *Symbolic Logic,* vol. 2 (1937), pp. 145–152.
6. – *Mathematical Logic,* Harvard Univ. Press, Cambridge, 1951.
7. – "Ontological Remarks on the Propositional Calculus", *Mind,* vol. 43 (1934), pp. 472–6.
8. – *A System of Logistic,* Harvard Univ. Press, 1934.
9. Rosenbloom, Paul C., *Elements of Mathematical Logic,* Dover Publications, Inc., 1950.
10. Russell, Bertrand, *An Inquiry into Meaning and Truth,* W. W. Norton and Co., Inc., N. Y., 1940.
11. Wajsberg, M., "Ein erweiterter Klassenkalkul" *Monatshefte für Mathematik und Physik,* Vol. 40 (1933), pp. 113–126.
12. Wittgenstein, Ludwig, *Tractatus Logico-Philosophicus,* Harcourt, Brace and Co, Inc., New York, 1922.

A theorem on permutations in models

by

LARS SVENONIUS
(University of Uppsala)

I. If S is any system in first-order logic (elementary system) with predicate constants R, Q, T_1, T_2, ..., and there is a model M of this system with a permutation φ of its elements which leaves the R-part of M unchanged but changes the Q-part, then we know that no definition of Q in terms of R, nor any disjunction of such definitions is provable in S. Theorem A, which will be proved here, is a converse of this. Theorem A′ may be regarded as a more syntactic formulation of the same proposition. The equivalence between A and A′ is easily seen.

Theorem A. Let S be any elementary system with the predicate constants R, Q, T_1, T_2, ..., such that no disjunction of explicit definitions of Q in terms of R is provable in S. Then there is a model M of S with a permutation Φ which is R-invariant but not Q-invariant.

Theorem A′. Let S be any elementary system with the predicate constants R (n-place), Q (m-place), T_1, T_2, ..., such that no disjunction of explicit definitions of Q in terms of R is provable in S. Then we may, without introducing a contradiction, add to the system a new 2-place predicate F, and the following axioms:

$$(x)(\exists y)(z)(Fxz \equiv z=y)$$
$$(x)(\exists y)(z)(Fzx \equiv z=y)$$

$$(x_1) \ldots (x_n)(y_1) \ldots (y_n)(Fx_1\, y_1. \ldots . Fx_n\, y_n. \supset Rx_1 \ldots x_n \equiv \equiv Ry_1 \ldots y_n)$$
$$\sim(x_1) \ldots (x_m)(y_1) \ldots (y_m)(Fx_1\, y_1. \ldots . Fx_m y_m. \supset Qx_1 \ldots x_m \equiv Qy_1 \ldots y_m)$$

II. It is necessary here to explain some terms which are used in the statement of theorem A, and some which will be used later.

The class of all objects in a model M, the "*universe*" of M, is denoted U_M.

A formula $A(x_1, \ldots, x_k)$ is called an $(R_1, R_2, \ldots)$-*condition* if it contains no extralogical constants except $R_1, R_2, \ldots$.

A one-one mapping φ between a subclass of U_{M_2}, and a subclass of U_{M_1}, is said to be $(R_1, R_2 \ldots)$-*invariant* if any two element-sequences s_1 and s_2 which correspond to each other by φ satisfy the same $(R_1, R_2, \ldots)$-conditions. The same terminology is used whether M_1 and M_2 are different or identical.

A one-one mapping with its domain and its converse domain equal to U_M is called a *permutation* in M.

A model M′ is called an *arithmetical extension* of M, if $U_M \subset U_{M'}$, and every sequence s of elements in U_M satisfies the same (first-order) conditions in M′ as in M.

III. A result which is closely related to theorem A, and which I will use in its proof, is the following theorem of Beth:[1]

T 1. Let S be a system with the predicate symbols R, Q, T_1, $T_2, \ldots$, and such that no definition of Q in terms of R is provable. Then it is possible to find two models M_1 and M_2 of S, with a one-one-correspondence φ between them which is R-invariant but not Q-invariant.

The proof of theorem A will be divided into three lemmas.

Lemma 1. Let S be an elementary system with the predicate constants R, Q, T_1, $T_2, \ldots$, such that no disjunction of explicit definitions of Q in terms of R is provable in S. Then we can find some model M of S with a one-one mapping φ between two subclasses D_1 and D_2 of U_M, which is R-invariant but not Q-invariant.

Proof: Since no disjunction of definitions is provable in S, we can find a complete extension S′ of S where the same is true. According to T 1, there are two models M_1 and M_2 of S', with a one-one mapping φ between them which is R-invariant but not Q-invariant. We let two different constant-sets $\{a_i\}$ and $\{b_i\}$

[1] See (1); also (2) and (4).

represent the elements in M_1 and M_2 respectively, and let $\Lambda_1(a_1, a_2, \ldots)$ be the set of sentences true in M_1, and $\Lambda_2(b_1, b_2, \ldots)$ the set of sentences true in M_2. The union

$$\Lambda_1(a_1, a_2, \ldots) \cup \Lambda_2(b_1, b_2, \ldots)$$

is consistent, and can be extended to describe a model M_3. In this model the mapping corresponding to φ has the desired properties.

Lemma 2. Let $M = \langle U_M, R, T_1, T_2, \ldots \rangle$ be any relational system, and let φ be an R-invariant one-one mapping between two subclasses D_1 and D_2 of U_M. Then it is possible to find some arithmetical extension M' of M with an R-invariant one-one mapping φ' such that the domain (alternatively: inverse domain) of φ' is U_M and such that φ' is an extension of φ.

Proof: We let the symbols $a_1, a_2, \ldots$ represent the elements of M and let $\Lambda(a_1, a_2, \ldots)$ be the set of all sentences true in M. Some of the symbols $a_1, a_2, \ldots$ correspond to elements in D_1; we denote them $c_1, c_2, \ldots$. We let $c_1', c_2', \ldots$ be the elements in D_2 which correspond to $c_1, c_2, \ldots$ by the mapping φ. The subclass of $\Lambda(a_1, a_2, \ldots)$ consisting of those sentences where R is the only predicate symbol may be denoted $\Lambda_R(a_1, a_2, \ldots)$. We now let $b_1, b_2, \ldots$ be a set of constants different from $a_1, a_2, \ldots$, and form the set $\Lambda_R(b_1, b_2, \ldots)$ by substituting, for every i, b_i for every occurrence of a_i in $\Lambda_R(a_1, a_2, \ldots)$. By $d_1, d_2, \ldots$ we denote the b's with the same indices as $c_1, c_2, \ldots$ among the a's. We let I be the set of all identities $d_i = c_i'$, and let J be the union

$$\Lambda(a_1, a_2, \ldots) \cup \Lambda_R(b_1, b_2, \ldots) \cup I.$$

If this set J is consistent, then it is possible to extend it so that it describes a model M', and in such a model the mapping φ': $a_i \longleftrightarrow b_i$ has the desired properties, i.e. it is R-invariant, it is an extension of φ, and its domain is equal to U_M. – By interchanging the roles of D_1 and D_2 in the construction above we would instead get a mapping with its converse domain equal to U_M.

It then only remains to be proved that the set J is consistent. If a contradiction were derivable, it would be derivable from a finite set, say from

$\{A_1(a_1, \ldots, a_r), B_1(b_1, \ldots, b_s), d_1=c_1', \ldots, d_k=c_k'\}$, where $A_1(a_1 \ldots, a_r)$ is a sentence in $\Lambda(a_1, a_2, \ldots)$ and $B_1(b_1, \ldots, b_s)$ is a sentence in $\Lambda_R(b_1, b_2, \ldots)$. By an elementary argument we can here eliminate those constants $a_1, \ldots, a_r$, which are different from $c_1', \ldots, c_k'$, and those constants $b_1, \ldots, b_s$, which are different from $d_1, \ldots, d_k$. We would then have a contradiction derivable from a set

$\{A_2(c_1', \ldots, c_k'), B_2(d_1, \ldots, d_k), d_1=c_1', \ldots, d_k=c_k'\}$, where $A_2(c_1' \ldots, c_k')$ is a sentence in $\Lambda(a_1, a_2, \ldots)$, and $B_2(d_1, d_2, \ldots)$ is a sentence in $\Lambda_R(b_1, b_2, \ldots)$. We know that R is the only predicate constant in the latter expression. However, by assumption the sequences $(c_1, \ldots, c_k)$, $(c_1', \ldots, c_k')$, and $(d_1, \ldots, d_k)$ satisfy the same R-conditions. It follows that the sentence $B_2(c_1', \ldots, c_k')$ is in $\Lambda(a_1, a_2, \ldots)$. From the consistency of $\Lambda(a_1, a_2, \ldots)$ follows then the consistency of the above set. Therefore J, too, is consistent. This concludes the proof of the lemma.

Lemma 3. Let $M_1, M_2, \ldots$ be a sequence of relational systems such that, for every $i=1, 2, \ldots$, M_{i+1} is an arithmetical extension of M_i. Then the sum $M=\cup M_i$ is an arithmetical extension of each M_i.

A proof of this lemma may be found in Tarski-Vaught (6), theorem 1. 9.

Proof of theorem A: To construct the desired model M with the desired permutation Φ we form a sequence of models $M_1, M_2, \ldots$ with mappings $\varphi_1, \varphi_2, \ldots$, such that the following holds.

(1) M_1 is a model of S with a one-one mapping φ_1, which is R-invariant but not Q-invariant.

(2) For every $k \geqslant 1$, M_{k+1} is an arithmetical extension of M_k with an R-invariant one-one mapping φ_{k+1} which is an extension of φ_k. When k is an odd number, the domain of φ_{k+1} is equal to U_{Mk}; when k is even, the converse domain of φ_{k+1} is equal to U_{Mk}.

The construction of the model M_1 is possible by lemma 1, and the construction of the succeeding models by lemma 2. According to lemma 3, the sum M of these models is a model, and the sum $\Phi=\cup\varphi_i$ of the mappings is easily seen to be a permutation

of M which is R-invariant but not Q-invariant. This concludes the proof of the theorem.

IV. I will finally indicate briefly some other consequences which can be drawn from the stated lemmas.

A permutation in a model M of S is called an *automorphism* if it is $(R_1, R_2, \ldots)$-invariant, and $R_1, R_2, \ldots$ are the only extra-logical constants in S. Under the same conditions we will call an arbitrary one-one mapping in M a *partial automorphism*. By using the lemmas 2 and 3 as in the above proof of theorem A, it is possible to prove the following:

T 2. Let M be some relational system with a sequence of partial automorphisms $\varphi_1, \varphi_2, \ldots$. Then there is an arithmetical extension M′ of M with automorphisms $\Phi_1, \Phi_2, \ldots$, which are extensions of $\varphi_1, \varphi_2, \ldots$

A finite relational system is itself its only arithmetical extension. This gives us the following corollary of T 2.

Corollary. In a finite relational system M every partial automorphism can be extended to an automorphism in M.

The following theorem will also be proved as a consequence of T 2.

T 3. Every elementary system S which admits infinite models has some model with an infinite group of automorphisms.[2]

Before stating the proof of the theorem, we will make some simple observations. A maximal set $\{A_i(x)\}$ of conditions in one variable x which are simultaneously satisfiable in a complete system S can be regarded as a maximal ideal in the Boolean algebra $B(S, x)$ which is formed by these conditions.[3] The set $\{A_i(x)\}$ is called a principal ideal, if there is some $A_i(x)$ such that $A_i(x) \supset A_j(x)$ is derivable from S for every j. The set $\{A_i(x)\}$ may be called *infinitely satisfiable* if it is possible to adjoin to S the sets $\{A_i(a_1)\}, \{A_i(a_2)\}, \ldots$ together with the

[2] This result is not new, but is contained in the stronger theorems 5.6 and 5.7 in Ehrenfeucht-Mostowski (3).

[3] Compare (5).

sentences $a_1 \neq a_2$, $a_1 \neq a_3$, ... $a_2 \neq a_3$, $a_2 \neq a_4$, ... without introducing a contradiction. The following lemma is easily proved.

Lemma 4. If S is a complete system and $\{A_i(x)\}$ is a maximal ideal in $B(S, x)$ which is not principal, then $\{A_i(x)\}$ is infinitely satisfiable.

Proof of T 3: The system S is assumed to admit infinite models. We let S' be some complete extension of S which also admits infinite models, and we consider the Boolean algebra $B(S', x)$ of its conditions $A(x)$ in one variable x. If this algebra is finite, then it has only a finite number of maximal ideals, and at least one of them must be infinitely satisfiable, since S' has an infinite model. On the other hand, if $B(S', x)$ is infinite, then it has some maximal ideal which is non-principal, and therefore (lemma 4) infinitely satisfiable. Let now $\{A_i(x)\}$ be a maximal ideal of $B(S' x)$ which is infinitely satisfiable. We form an extension S'' of S' by adjoining the sets $\{A_i(a_1)\}$, $\{A_i(a_2)\}$, ... together with the sentences $a_1 \neq a_2$, $a_1 \neq a_3$, ... $a_2 \neq a_3$, ... This extended system S'' is consistent and has therefore a model M. In this model, we have the following system of partial automorphisms: φ_1: $a_1 \longleftrightarrow a_2$, φ_2: $a_1 \longleftrightarrow a_3$, φ_3: $a_1 \longleftrightarrow a_4$, ... According to T 2, there is an arithmetical extension M' of M with automorphisms Φ_1, Φ_2, ..., which are extensions of φ_1, φ_2, ... It is easily seen that Φ_1, Φ_2, ... must all be different. M' is therefore a model of S', and of S, with the desired properties.

REFERENCES

1. Beth, E. W.: On Padoa's method in the theory of definition. *Indagationes Mathematicae, 15 (1953).*
2. Craig, W.: Three uses of the Herbrand-Gentzen theorem. *Journal of Symbolic Logic, 22 (1957).*
3. Ehrenfeucht, A. and Mostowski, A.: Models of axiomatic theories admitting automorphism. *Fundamenta Mathematicae, XLIII (1956).*
4. Robinson, A.: A result on consistency and its application to the theory of definition. *Indagationes Mathematicae, 18 (1956).*
5. Svenonius, L.: $\aleph_0$-categoricity in first order predicate calculus. *Theoria, XXV (1959).*
6. Tarski, A. and Vaught, R. L.: Arithmetical extensions of relational systems. *Compositio Mathematica, 13 (1957).*

DISCUSSION

The Problem of experience in the Gestalt Psychology.

By *Lajos Székely* (Stockholm).

Pro captu lectoris habent sua fata libelli.

Terentianus Maurus

I

When the work which a person does is successively improved – for example, the diagnostics of a physician, we explain this improvement as the result of experience. This is the explanation given by persons with no professional knowledge of psychology; and, in principle, the same explanation is given by the scientifically trained psychologist. In this connection, "experience" is a word used to explain certain phenomena; but at the same time, experience remains something about which we know very little. In reality, what do we know about what experience is or how it is acquiered or built up? How does scientific psychology approach the problem of experience?

As is well known, scientific psychology, or the study that is referred to by that name, is not a uniform science with a coherent theoretical basis. It is divided into various branches, where the problem of experience is formulated very differently. In this paper, I wish to consider in what way the problem of experience is, either implicitly or explicitly, dealt with by learning psychology and Gestalt psychology. It should be pointed out, however, that ex officio, the problem of experience may be said to have its right of domicile in learning psychology, whereas it became naturalized only later in Gestalt psychology.

Let us take as our starting-point an event that has occurred spontaneously, that is to say, without the express design of an experimenter, and observe how the representatives of the different lines of thought envisage, in this connection, the problem of experience.

> Peter, who is four years old, is hammering a nail into the door of a barn. When he has done this, he takes a second nail and wants to hammer it into another part of the door where the iron-fitting is. He finds that he can't do it. Peter's face expresses

anger; but suddenly he realizes what is happening, and says: "It's not possible because this", and he points to the iron-fitting, "is made of stone." He then proceeds to busy himself with something else.[1]

In another nursery school, Paul, who is the same age, is also playing with nails. After successfully hammering a nail into the woodwork, he now tries to hammer one into the iron-fitting. Here, Paul likewise meets with the hard resistance of the material. He also becomes irritated and discouraged. He throws the hammer on the floor. Picks it up again, and tried to hammer the nail into another part of the iron-fitting. This time, he throws the nail away, and shouts, "silly nail, it doesn't want to go in." He then takes another nail and makes a similar attempt. He continues his efforts in this way; then becomes quite angry; hammers the door with his fists, stamps on the hammer, and kicks a boy, who is passing, on the shins.

What attitude does *learning theory* adopt to these observations? Actually, spontaneous observations do not come within its sphere, because they comprise too many and too few controllable variables. But no particular difficulties arise when interpreting the observed events from the point of view of learning theory. A smaller number of experiences (or experiences of shorter duration) proved sufficient for Peter to learn by, and to change his behaviour. Paul, on the other hand, needed a larger number of experiences (or experiences of longer duration) in order to learn. It is not explicitly stated, but it is implicitly assumed, that under similar conditions, both Peter and Paul *have the same experience*. The difference lies in the required number (or duration) of the individual experiences.

The events which have been previously described, would be regarded by *Gestalt psychology* as thought processes (the solution of problems), and would be interpreted as indicating that Peter had understood (realized) why he could not hammer the nail into the iron-fitting, but that Paul had not. In a subsequent experiment, and under different conditions, tests would be carried out to ascertain how Peter and Paul solve problems; and it would be shown that the understanding exhibited in the previous situation was related to the solution of problems in the later situation. Here, it would be stated explicitly that different thought processes had taken place (in connection with the hammering) in Peter and in Paul; and that consequently, different thought processes were also to be expected in the subsequent situation comprising the solution of problems.

[1] This observation is recorded in a book by Isaacs (2), and is here slightly modified.

It is strange that the two events:

"Peter tries to hammer in a nail, and fails"
"Paul tries to hammer in a nail, and fails"

represent (FOR THE STUDENT OF THE PSYCHOLOGY OF THINKING) different thought processes, whereas (FOR THE STUDENT OF LEARNING PSYCHOLOGY) they represent an identical experience. But common sense would seem to indicate that *thought processes develop into experiences. In that case, however, Peter and Paul have not had the same experience.* That Peter learns more quickly and with less experience is not the only, and perhaps not the most important differences between them; but the essential point is, that the same occasion enables Peter to *comprehend* something, that Paul does not understand; and consequently, *Peter's experience is different from Paul's.*

We can now formulate the problem of experience as it is *explicitly* stated in the works of the Gestalt psychologists (Koffka, 5; Katona, 4; Székely, 10): (1) We distinguish between the *occasion* on which a living creature acquires experience, and (2) the *experience itself.* On the *same occasion,* Peter and Paul gained *different experiences.* We distinguish between (3) the *experience-work* and (2) the experience acquired; just as in the psychology of thinking, a distinction is made between the thought-work (thought-process, solving-process) and the thought,[2] or between the insight-work and what is understood; or the distinction that is made in psychoanalysis between the dream-work and the dream, between the joke-work and the joke, or between the pathogenic processes that lead to the genesis of a symptom, and the symptom itself. In the case of Peter and Paul the experience-work was different: in Peter's case, this consisted in an insight-work, whereas in regard to Paul, it was something different; perhaps disappointment, anger, failure, impotence, etc. Consequently the experience gained was also different.

In the work of Székely (11), in a section entitled "Dependence of availability upon intellectual events previously concluded", seven studies are mentioned. On reading these studies, it will be observed, that in the first, the experiences are described as "solving-work"; in the second, as "insight-work" (understanding); and then as "memorizing", etc.;

[2] The terminology is not always so uniquivocal. The expression "problem-solving" refers to both the process and the final solution. In most of the experiments, the solution consists in an instruction, e.g. in equilibrium experiments: light a candle, etc. But if the descriptions are read carefully, one cannot fail to observe where the solving-process is described, and where the solution obtained is presented.

and, moreover, the effects were investigated, which various modes of acquiring experience (experience-work), exerted on subsequent solving-work, etc.

The distinctions previously mentioned are foreign to learning psychology with its traditionally different approach. In learning psychology, "experience" is defined in terms of the "occasion"; and it is tacitly assumed that the same experiences are either acquired or built up on the same occasions. Differences in the time needed for acquiring the experience, or differences in the number of individual experiences, are then considered in relation to the subsequent effects.

II

In a recent work. Saugstad (9) makes grave charges against Gestalt psychology: "The second weakeness in the Gestalt framework is found in the highly inadequate treatement of the role of past experience in problem-solving. The Gestalt psychologists have failed to make provisions for the role of experience in problem-solving. Of course, they do not deny that experience plays a role in problem-solving, *but none of them has made a serious attempt to formulate explicitly the role played by experience.*" (My italics.) An again: "Gestalt oriented psychologists, such as Székely, have not profited as much from the . . . experiments on learning as would seem reasonable to expect."

All that has been stated above is a direct refutation of Saugstad's contention. For the sake of mutal understanding I should like to investigate how he could "pass over" "experience" in reading the works of the Gestalt psychologists. As is well known, to understand, presupposes two persons: one who communicates something, and the other who understands it. According to the technical language employed in the theory of communication, somewhere a "noise" must be present which prevents Saugstad from reading precisely that which the authors have written in their publications.

I could well imagine that the deviations from the prevalent tradition, which are to be found in the approach of the Gestalt psychologists in regard to the presentation of the problem, the language used, and a strange result, constitute the "noise" in Saugstad's case.

(A) When Köhler and Wertheimer began their studies in the psychology of htinking, the philosophical tradition of *empiricism* exerted a powerful influence on experimental psychology. We need only recall the animated discussions on the questions whether the perceptual constancies and the space values of retinal points, etc., were innate or acquired from experience. The empiricist tradition of thinking had not investigated the fact of experience; but had used it as an *explanatory concept*. Köhler and Wertheimer were justified in opposing the em-

piricist tradition of thinking. In doing this, they did not deny the actual fact of acquiring experience or its importance. What they did, was to avoid in the theoretical discussions, the misuse of a word which represented too few known data. Perhaps this "perspicacity" and "incorruptibility" of Gestalt psychology in respect of theory, is confused with "blindness for the phenomenon"; and consequently, Saugstad believes that the significance of experience for thinking is disregarded.

(B) The Problem. As we have seen from what has been previously stated, in Gestalt psychology a clear distinction is made between experience-work, experience, and the objective occasion. In traditional learning psychology, however, this distinction is not made. For the most part, in the experimental investigations of the Gestalt psychologists, the problem has been formulated as follows: what effect do the various types of experience-work exert on subsequent thinking. In many studies the experience-work consists in the understanding of a natural phenomenon, or in the understanding of a physical demonstration. In some experiments it consists in the work performed in memorizing, whereas in other experiments again, it refers to the work that leads to the solution of intellectual problems. The language employed in Gestalt psychology for descriptive purposes, deviates widely from that used in accordance with the prevalent tradition in learning psychology. In traditional learning psychology, with its emphasis on the *objective occasion,* experience is described as "choice", "conditioning", "reaction", "memorizing", etc. In Gestalt psychological investigations the process of solution or similar phenomena are described in detail. If a completed process of solution is subsequently considered in relation to a later process of solution (irrespective of whether this occurs days or weeks later), the former is then regarded as having become an experience. But anyone who is accustomed exclusively to the terminology of the classical theory of learning, may possibly not be able to recognize that – in respect of the subsequent event – the effect of an experience was here investigated.

Köhler (6), Wertheimer (12) and Duncker (1) investigated the thought process in its *structural aspect.* A work (10), however, which also has the title: "Zur Psychologie des inneren Verhaltens beim Lernen, Denken und Erfahren" (On the Psychology of the Internal Procedure in Learning, Thinking and Experiencing), deals explicitly with the problem: what relation exists between problem-solving events, and between understanding-work and problem-solving, etc., which are separated from one another in time. The latter problem is termed the *historical aspect* of the investigation. The author concludes that none of these aspects may be omitted when investigating cognitive processes. The framing of the problem and the terminology must indeed create a

very loud "noise" from the point of view of the theory of communication, when a reader fails to notice experience is dealt with here.

(C) Results. The third reason why Saugstad believes that the Gestalt psychologists either deny or undervalue the role of experience in thinking, is due, in my opinion, to the results obtained by these psychologists. Wherever the Gestalt psychologists investigate the effects of past experience on subsequent cognitive processes, they compare the effect of past memorizing with the effect of past problem-solving-work understanding-work (intelligent learning). The results of the comparison are unfavourable to memorizing. Moreover, in regard to problems, whose solution requires a certain degree of originality, *memorizing is shown to be quite ineffective; but this does not apply to intelligent learning.* I will readily admit that this result is not particularly original, since this has been known for at least 2,000 years. Nevertheless, I believe that for some professional psychologists this may prove a barrier to understanding.

During the course of their lives, people acquire experience in a great variety of ways, which are too numerous to survey. Traditional learning psychology has taken only a very limited number of these ways as a standard for its experimental models. The majority of these models are based on mere memorizing and stereotyped practice. We do not know (and, moreover, this question has not received attention) if precisely these methods represent the most important modes of acquiring experience in life. Under such conditions, when *humane experience* becomes equated with the model of memorizing, a psychologist can possibly interpret the work of the Gestalt psychologists to mean that very little importance is attached to the effect exerted by experience on thinking.

III

Once the suspicion was created that the Gestalt psychologists are heretics who deny experience, it becomes possible to detect heresy everywhere, even when quite a different subject is being discussed. In the essay (11) referred to, I dealt with a metatheoretical question, namely, concept formation in Gestalt psychology and in the study by Saugstad and Raaheim. It was indicated there, that the concept of "availability" in the work (8) is regarded as a *property*, whose presence may be determined by a test. Problem-solving is then explained as the result of this "property". The Gestalt psychologists consider that the availability "is the outcome of previous events . . ."; and that problem-solving can be explained functionally.

Since Lewin (7) and Kaila (3) have discussed, in considerable detail, the question of concept formation in psychology, I felt justified in

assuming that my readers would, to some extent, be acquainted with this subject. The omission will now be made good by giving two examples to illustrate what is meant both by functional explanation and by explanation in terms of a property.

Fact: "Wood floats."
Explanation (A): "Because it possesses the property of floating."
Explanation (B): By means of the hydrostatic law.

Fact: "Pelle suspects that his wife is deceiving him. Since the newspapers were full of the conflict in the Far East, he believes that the newspapers report that she is deceiving him with an Oriental."

Explanation (A′): "Because Pelle suffers from a pathological form of jealousy."

Explanation (B′): With the aid of psychoanalytical theory, on the basis of the description of his infantile conflicts, his pathogenic defense organization which had been built up by him for the solution of his conflicts; and the role of the actual conflicts.

A and A′ represent explanations in terms of a property; and B and B′ are functional explanations. It appears rather far-fetched to take a metatheoretical discussion as a proof that Székely does not believe in the effect exerted by experience on thinking.

IV

We psychologists have been assigned a very difficult task. We are expected to use our knowledge in the service of "psychological strategy" that aims at promoting international understanding. But even intra muros, it is extremely difficult for those who have been brought up in different "schools" to understand one another.

REFERENCES

1. Duncker, K.: *Zur Psychologie des produktiven Denkens.* Berlin: Springer, 1935.
2. Isaacs, S.: *Intellectual Growth in Young Children.* London: Routledge & Kegan Paul, 1930.
3. Kaila, E.: Physikalismus und Phänomenalismus. *Theoria,* 1942, 8, 85–125.
4. Katona, G.: *Organizing and Memorizing.* New York; Columbia Univ. Press. 1940.
5. Koffka, K.: *Principles of Gestalt Psychology.* London: Routledge & Kegan Paul, 1935.

6. Köhler, W.: *The Mentality of Apes.* London: Routledge & Kegan Paul, 1925.
7. Lewin, K.: Der Übergang von der aristotelischen zur galileischen Denkweise in der Biologie und Psychologie. *Erkenntnis,* 1930, 1.
8. Saugstad, P. & Raaheim, Kj.: Problem-Solving and Availability of Functions. *Nord. Psykol.* 1957, 9, 205–220 and *Acta Psychol.* 15, 263–278.
9. Saugstad, P.: Availability of Functions. A Discussion of Some Theoretical *Aspects. Nord. Psykol.* 1958, 10, 216–232 and *Acta Psychol.*
10. Székely, L.: Zur Psychologie des inneren Verhaltens beim Lernen, Denken und Erfahren. *Theoria,* 1947, 13, 157–182.
11. – Some Comments on Problem-Solving, Availability and Test-Magic. *Nordisk Psykol.* 1958, 10, 108–113 and *Acta Psychol.* 14, 152–157.
12. Wertheimer, M.: *Productive Thinking.* New York: Harpers, 1945.

REVIEW

Naess, Arne: *Wie fördert man heute die empirische Bewegung? Eine Auseinandersetzung mit dem Empirismus von Otto Neurath und Rudolph Carnap.* Oslo, 1956, Universitetsforlaget. Pp. 48, Mimeographed, NCr. 9.75. By *Eivind Storheim* (University of Oslo).

William James once compared the system of Hegel to a crowded hotel in which no rooms are unoccupied. The universe of Hegel contained no secrets. The "Weltgeist" permeates everything. This rigid monism with no possibility for the unknown, this idea of a universe in which nothing remains to be discovered, made James philosophically sick and led him to abandon Hegel's system altogether.

It resulted in James' adoption of a pluralistic philosophy in which there was room for unexplored possibilities.

I think that something like this has happened to Næss in relation, not to Hegel, but to the logical empiricists of the nineteen–thirties. His small book contains a searching criticism of fundamental points in the physicalistic doctrines of Neurath and Carnap, the two foremost champions of logical empiricism at that time.

Næss' book is written in the years 1937–39, but for reasons touched in the preface, not published till now. This is to be regretted as it is written by a critic with first hand knowledge of the methods and doctrines of the logical empiricists (Næss had studied in Vienna in the early thirties). The book thus belongs to another category than the criticism that came from more "metaphysical" quarters. As this criticism was very often misdirected or excessivly polemical it was not seriously met. Næss' book, however, might have stimulated discussion resulting in clarification of the basic commitments of logical empiricism. To-day, after twenty years, the book has lost some of its actuality, as there no longer exists a definite group of philosophers advocating the views criticized. But the issues are fundamental and still very much alive.

Næss starts his criticism with a statement to the effect that the usual definitions and uses of terms like "empirical", "positivism", "ex-

perience" are so vague and indefinite that they are useless for the purpose of classifying texts or authors with any certainty. Because of this vagueness it is rather arbitrary who is labelled "positivist" and who "metaphysician". The function of such hazy definitions and careless terminology is to create the impression in unwary or untrained readers that violent disagreements or great differences exist between two thinkers because the one is called "positivist" and the other "metaphysician". The effects are detrimental to philosophy because patient inquiry into the more fundamental point of divergence between different thinkers and the elaboration of more exact and fruitful concepts for classification of doctrines are hampered. The logical empiricists as well as their antagonists were both committed to this looseness in terminology.

Although the logical empiricists used terms like "metaphysician" as a slogan, they also tried to work out a criterion by help of which they should be able to classify a given statement as metaphysical in a more precise sense. It was generally agreed that whatever else a metaphysical sentence was, it was devoid of meaning. The adoption of the "verification principle of meaning" gives at least partly, the background to the thesis of physicalism.

In the earlier phase of logical empiricism, the phenomenalistic stage, a sentence was said to have meaning if and only if all its terms, except the logical ones, referred to sense-data. The task intended to be done by this definition of meaning was that any meaningful sentence in principle was *verifiable* by other persons. If someone says "There is a red car in the street" (P) this sentence was meaningful because anyone was thought to be able to verify it by means of sense-data. But as sense-data are not floating in the air, but always relative to a given person it was soon shown that this definition of meaning did not do what it was expected to do. How can anyone be sure that his sense-data are identical with somebody else's? If you say P how can *I* or anyone else verify it if its meaning consists in *your* sense-data? The remedy sought for this dilemma was that there was a structural identity between P as uttered by one person and P as uttered by another. This structural identity consisted in the form of the statement being identical although the content might not be. This solution was, however, not convincing as the distinction between form and content was left unclear. Phenomenalism did thus not satisfy the verification principle.

Out of this situation grew the thesis of physicalism. According to this doctrine basic statements like "I see a red car now" referred directly to intersubjectively observable phenomena in space and time. The sense-data were now, it seems, not to be taken as the ultimate source of knowledge *per se* but only as reports about real phenomena

in space and time. This thesis was not supported by any arguments except an appeal to commonsense and the practise of various empirical sciences. The problem of verification was now "solved" by the supposition that physical objects were "there" to be observed by anyone. All meningful statements could thus be expressed in the language of physics.

As compared with phenomenalism the thesis of physicalism represents a step backwards in philosophical acumen. But it laid the methodological foundation for the unity of science movement. The language of physics was proclaimed to be the universal language of science. Every meaningful statement was to be expressed in that language. All phenomena, physical, psychological, sociological were to be described in physical terms and located by spacetime coordinates.

It is this all-devouring monism that Næss criticizes. And for two reasons. Firstly it is not clear what is, according to the physicalists, to count for a "thing". The verificational principle of meaning in its physicalistic shape does not give us a clear criterion by which we can distinguish metaphysical from not-metaphysical sentences. This is because of the vagueness of the expression "thing-language" used by Carnap to refer to our statements of things in space and time. Some of Næss' arguments in this connection are not quite fair. He asks, for instance, what the physicalists mean by the expression "language of physics" which is central to the doctrine. Having noted that Carnap sometimes uses the term "language" in the technical sense of a calculus consisting of formation and transformation rules, Næss infers that "language" is also used in that sense in the expression "language of physics". Where, Næss asks, is such a language elaborated? I do not think it is plausible to interpret "language" in the context "language of physics" as Næss does. Carnap was certainly aware of the fact that only small parts of physics (perhaps only the mathematical) consisted of a language in that sense. It was rather the program of physicalism to elaborate such a language. That the principles for this elaboration were unclear is another matter.

Also, Næss cites a textbook in chemistry in which he finds the sentence "Methan occurs in volcanic gases", and he asks whether "occurs" is a scientific term coming under the rules of chemistry. Now, this is a weak argument because the physicalists did not claim that all terms in scientific books were regulated by explicit rules. What they claimed was that all expression in order to be scientific should be reducible in principle to the thing-language. That this reduction could easily be done with the term "occurs" is obvious.

These, however, are minor points. The chief reason for which Næss rejects physicalism is that in his opinion it represents a serious impediment to research. It represents a dogmatic attitude, in his eyes, that

a sentence in order to be scientifically respectable shall be reducible to physical terms. Can we be quite certain that the conceptual framework of physics, with its concept of spacetime as fundamental, is the only adequate and fruitful frame of reference for our inquiries into all phenomena that surround us? Will not a rigorous application of the reduction principle exclude us from investigating problems, which initially are vague, but could be made more definite? According to Næss the physicalists should not have propounded their theses with an absolutistic claim. Physicalism could have been exciting as a proposol for discussions and inquiry. But it should not *a priori* exclude us from certain types of methods and inquiry which only research could prove to be fruitful or not. Physicalism would, stripped of its dogmatic tiara, not have claimed that the only legitimate language of research was the language of physics, but only: Let us try how far we can get when we formulate the sentences within a given field of research in physical terms. Næss thus gives priority to research areas not to any specific "scientific" method. As an example that shows how preconceptions about method can block research, Næss refers to certain psychologists who were unwilling to take up research in the field of parapsychology because the subject did not fit into the existing methods and framework of traditional psychology.

That logical empiricism with its emphasis on the use of mathematical logic as a tool for analysis easily neglected problems existing in connection with the transition from natural languages to logical calculi (or vice versa), is well exposed in a note (p. 37/8) on Carnap's article "Testability and Meaning" (*Philosophy of Science*, Oct. 1936, p. 464).

Carnap wants here to prove that a number of plausible interpretations, T_1, T_2 ... of a given sentence T_0 are cognitively heteronymous with T_0.

T_0 runs: "On May 6, 1935, at 4 P.M., there is a round black table in my room". T_1 runs: "If on May ... somebody is in my room and looks in such and such direction, he has a visual perception of such and such a kind".

It is Carnap's proof for the heteronymity relation obtaining between T_0 and T_1 that Næss criticizes.

Carnap proceeds as follows:

T_1 is (according to Carnap) a universal implication sentence which can be formulated thus:

"(x)((x is ... in my room and looks ...) ⊃ (x perceives ...))" which can be abreviated to

$$(3) \qquad (x)\,(P(x) \supset Q(x))$$

By the help of calculatory operations (3) is easily transformed into (4) $(x)(\sim P(x) \vee Q(x))$.

Now, in the case there is neither a black table in my room nor an observer, T_0 will be false as will also P(x).

(3) and (4), however, will be true. Hence, T_1 is proved to be cognitively heteronymous with T_0.

There are several steps in this proof that Næss questions. How can Carnap be certain that "if – then" in T_1 is equipollent with the horse shoe sign "$\supset$" in a given calculus? What hypothesis concerning "if – then" permits Carnap to transform T_1 into (3)? And how can he be sure that (3) and (4) in their de-symbolized shape are equivalent? According to Næss, Carnap implicitly accepts certain hypotheses about actual usage, but this is slurred over. His proof thus looks logically neat, free from all empirical commitments (For a detailed analysis of the problems here involved, see Jacob *Meløe:* "Dialogue on the Hypothetical Character of Logical Analysis", *Inquiry*, Vol. 1, number 1, 1958 p. 77 ff.).

It is a question, however, whether Næss' criticism is relevant to Carnap's intentions. If Carnap only wants to show that all *symbolized* interpretations of T_0 are cognitively heteronymous with, i.e. do not have the same truth-values as, T_0 *symbolized,* his proof is all right. However, it is to be suspected that Carnap's claims are not that modest. It seems that he means that by the help of logical calculi one can prove relations between sentences in natural languages. If so, he has overlooked a genuine philosophical problem in his enthusiasm for symbolizations.

Enough has been said to indicate Næss' criticism. He criticizes the physicalists in the name of a radical empiricism. His sloganlike formulation runs: Not "science" but research.

But, however open-minded one wants to be there is need for certain rules to guide ones research. Næss does not discuss what these are. His criticism is thus positive only in stating the wide field of problems that can be investigated without any prejudices, not in the sense of giving an alternative to the fundamental methodological and ontological views of the physicalists. If the ghost of materialism was lurking in the background of physicalism, Næss seems to be committed to a pluralistic ontology. This, however, is unfortunately not explicitly stated. A further elaboration of what is to be counted as "experiences", their ontological status and classification, which constitute the basic problems of a radical empiricism which also is pluralistic, would have been welcome. It is to be hoped that the book may contribute to discussion on these fundamentals.

SCANDINAVIAN BIBLIOGRAPHY (XXXIX)

(January 1st to December 31st, 1958)

DENMARK

Christensen, Johnny: *Actus purus. An essay on the function and place of pure act in Aristotelian metaphysics and on its interrelation with some other key concepts.* Classica et Mediaevalia XIX; p. 7–40.

Diderichsen, Børge: *Sandhed og existens.* [*Truth and existence.*] Tidehverv XXXII; p. 1–7.

Diensen, Palle: *Linier i dansk systematisk teologi efter krigen.* [*Trends in Danish theology after the war.*] Nordisk Missionstidsskrift LXIX; p. 18–28.

Eklund, Harald: *Religionen skapar problem. En orientering i religionsfilosofi.* [*Religion creates problems. An orientation in the philosophy of religion.*] Gyldendal, Copenhagen 1958. 65 p. 9,– DCr. (=Scandinavian University Books.)

Gelsted, Otto: *Den graeske tanke.* [*Greek thought.*] Thaning & Appel, Copenhagen, 1958. 190 p. 16,– DCr.

Gjessing, Kjetil: *Noen refleksjoner over kunst og intellektualisme.* [*Some reflections on art and intellectualism.*] Vinduet XII; p. 136–143.

Hartnack, Justus: *Logik. Klassisk og moderne.* [*Classic and modern logic.*] Gyldendal, Copenhagen 1958. 142 p. 14,75 DCr.

Hillerdal, Gunnar: *Teologisk och filosofisk etik. Brytningar och synteser i etikens historia från antiken till nutiden.* [*Theological and philosophical ethics. Controversies and syntheses in the history of ethics from antiquity to our days.*] Gyldendal, Copenhagen 1958. 273 p. 36,50 DCr. (=Scandinavian University Books.)

Jaspers, Karl: *Udødelighed.* [*Immortality.*] Vindrosen V; p. 233–238.

Johansen, J. Prytz: *Komplementaritet og kulturhistorie.* [*Complementarity and the history of culture.*] Perspektiv V, 3; p. 22–27.

Livsanskuelse gennem tiderne. Bd. 4. Renaissance. I. Erasmus af Rotterdam, Pomponazzi, Galilei. [*View of life through the ages. Vol. 4. I. Erasmus, Pomponazzi, Galilei.*] Ed. by Aage Marcus & B. Ochsner. Gyldendal, Copenhagen 1958. 121 p. 12,75 DCr.

Livsanskuelse gennem tiderne. Bd. 10. Det 20. århundrede. Freud, Lenin, Schweitzer, Macneill, Dixon, Heisenberg. [*View of life through the ages. Vol. 10. The 20th century. Freud, Lenin, Schweitzer, Macneill, Dixon, Heisenberg.*] Ed. by Aage Marcus & B. Ochsner. Gyldendal, Copenhagen 1958. 139 p. 12,75 DCr.

Livsanskuelsernas brydning i vor tid. Fem foredrag. [*Today's controversies on views of life. Five lectures.*] By Börge Diderichsen, Heinrich Roos, Jörgen Jörgensen, Paul Diderichsen and J. Th. Arnfred. Det danske Forlag, Copenhagen 1958. 115 p. 9,75 DCr.

Løgstrup, K. E.: *Videnskap og livstydning.* [*Science and interpretation of life.*] Vindrosen V; p. 148–155.

Madsen, K. B.: *Komplementaritet i psykologien.* [*Complementarity in psychology.*] Nordisk Psykologi X; p. 1–14.

Madsen, K. B.: *Terminologiproblemet. Psykologisk terminologi og systematik.* [*The problem of terminology. Terminology and system in psychology.*] Nordisk Psykologi X; p. 281–284.

Vaščenko, Hryhori: *Den materialistiske oppfatning av »mennesket» i Sovjet.* [*Materialistic conception of "man" in Soviet Russia.*] Samtiden LXVII; p. 166–170.

Wiśniewski, Bohdan: *The problem of cognition of the external world in Stoic philosophy.* Classica et Mediaevalia XIX; p. 1–6.

Witt-Hansen, Johannes: *En kritisk analyse av materiebegrebet hos Newton, Kant og Einstein.* [*A critical analysis of the concept of matter in Newton, Kant and Einstein.*] Gad, Copenhagen 1958. 108 p. 12,– DCr.

Witt-Hansen, Johannes: *Exposition and critique of the conceptions of Eddington concerning the philosophy of science.* Gad, Copenhagen 1958. 125 p. 12,– DCr.

FINLAND

Aaltonen, Jorma: *Epäily intohimona.* [*Doubt as passion.*] (Review of: Alan Wood: Bertrand Russell, the passionate sceptic.) Suomalainen Suomi 1958, 6; p. 393–400.

Ahmavaara, Yrjö & Markkanen, Touko: *The unified factor model.* Helsinki, Väkijuomakysymyksen tutkimussäätiö. 187 p. 900,– FM. (=Alcohol research in Northern Countries.)

Ahmavaara, Yrjö: see Vahervuo, Toivo.

Ajatus XX. Filosofisen yhdistyksen vuosikirja. [*Thought. Yearbook of the Philosophic Society.*] Distributor: Akateeminen Kirjakauppa, Helsinki. 273 p. 600,– FM.

Contents:

Harva, Urpo: *Psykoanalyysin meta-analyysi.* [*Meta-analysis of psychoanalysis.*] p. 5–26.

Hintikka, Jaakko: *Arvokäsitteistä sosiaalitieteiden metodiopissa.* [*On value concepts in the methodology of the social sciences.*] p. 27–48.

Hintikka, Jaakko: *Modality as referential multiplicity.* p. 49–64.

Hintikka, Jaakko: *Necessity, Universality and Time in Aristotle.* p. 65–90.

Ketonen, Oiva: *Oppikoulukysymys ajankohtaisena kulttuuripoliittisena ongelmana.* [*The secondary school problem as an actual problem of cultural politics.*] p. 91–106.

Tenkku, Jussi: *Moorali ja rikkaus onnen perusteina. Tutkimus Adam Smithin moraalifilosofiasta.* [*Moral and wealth as grounds of happiness. A study on A. Smith's moral philosophy.*] p. 107–258.

Tenkku, Jussi. *Tosiasia- ja arvoarvostelma Humen filosofiassa.* [*Judgment of fact and judgment of value in Hume's philosophy.*] p. 259–273.

Ajatus XXI. Filosofisen yhdistyksen vuosikirja. [*Thought. Yearbook of the Philosophical Society.*] Distributor: Akateeminen kirjakauppa, Helsinki. 121 p. 400,– FM.

Contents:

Kaila, Eino: *Einstein-Minkowskin invarianssiteoria.* [*The Einstein-Minkowski invariance theory.*]

Allardt, Erik: *Viisaus ja koeteltavuus. Nykypäivien sosiologian ongelmia.* [*Wisdom and testability. Problems of sociology today.*] Suomalainen Suomi 1958, 3; p. 137–140.

Ames, Louise Bater: see Gesell, Arnold.

Berefelt, Gunnar: *Den objektiverade känslan. En kommentar till Susanne Langers poesiteori.* [*The objectified sentiment. A comment on S. Langer's theory of poetry.*] Nya Argus 1958, 17; p. 257–259.

Bergson, Henri: *Henkinen tarmo.* [*"L'énergie spirituelle."*] Transl. by J. A. Hollo. Porvoo-Helsinki, Werner Söderström. 208 p. 350,– FM. (=Humanitassarja 8.)

Bonduelle, André: *Tuomas Akvinolaisen etiikasta. Vastaus tri Olavi Castrénille.* [*On the ethics of Thomas Aquinas. A reply to O. Castrén.*] Teologinen aikakauskirja 1958, 1; p. 47–51. With a reply of Olavi Castrén, p. 51–54.

Chydenius, Johan: *The typological problem in Dante. A study in the history of mediæval ideas.* Helsinki, Societas Scientiarum Fennica. 159 p. 800,– FM. (=Commentationes Humanarum Litterarum XXV, 1.)

Emerson, Ralph Waldo: *Esseitä.* [*Essays.*] Transl. by J. A. Hollo. Porvoo-Helsinki, Werner Söderström. 399 p. 450,– FM. (=Humanitas-sarja 4.)

Fieandt, Kai von: *Helsingin paljasjalkaiset ja uusi asuttajapolvi.* [*Natives of Helsinki and the new inhabitants.*] Helsinki, Helsingin kaupunki. 110 p. 700,– FM. (=Helsingin kaupungin julkaisuja n:o 7.)

Gaier, Eugene L.: *Sosiaalipsykologisia virtauksia.* [*Currents of*

social psychology.] Transl. by Pirkko Mela. Suomalainen Suomi 1958, 4; p. 153–156.

Gesell, Arnold & Ilg, Frances L. & Ames, Louise Bater: *Ungdom. Åren tio till sexton.* [*Youth. The years from ten to sixteen.*] Transl. by Karl Hylander. 500 p. 2208,– FM. (=Scandinavian University Books.)

Haavio, Martti: *Eino Kaila ja arvot.* [*E. K. and values.*] Parnasso 1958, 8; p. 344–353.

Haavio, Martti H.: *Puoli vuosisataa Eduard Sprangerin pedagogista elämäntyötä.* [*Half a century of E. Spranger's pedagogical life work.*] With English summary. Kasvatus ja koulu 1958, 1; p. 1–9.

Haikola, Lauri: *Uskonto ja totuus. Uusinta ruotsalaista uskonnonfilosofian keskustelua.* [*Religion and truth. Modern discussion on the philosophy of religion in Sweden.*] Teologinen aikakauskirja 1958, 5; p. 249–265 & 6; p. 340–352.

Harva, Urpo: *Etiikka.* [*Ethics.*] Porvoo-Helsinki, Werner Söderström. 259 p. 650,– FM.

Heikkinen, Väinö: *A study in the learning process in the school class environment.* Helsinki, Academia Scientiarum Fennica. 54 p. 260,– FM. (=Annales Academiae Scientiarum Fennicae. Ser. B. Tom. 111,3.)

Helanko, R.: *Theoretical aspects of play and socialization.* Turku, University of Turku. 48 p. 200,– FM. (=Annales Universitatis Turkuensis. Ser. B. Tom. 70.)

Hollo, J. A.: *Huoli humanismista* [*Care of humanism.*] (Review of G. H. von Wright: Logiikka, filosofia ja kieli.) Suomalainen Suomi 1958, 4; p. 251–255.

Honkasalo, Brynolf: *Oikeuden ja tahdonväpauden käsitteiden kehityspsykologinen arviointi.* [*Review of the concepts of justice and freedom of will from the point of view of evolutional psychology.*] Lakimies 1958, 2; p. 116–124.

Honkasalo, Brynolf: *Perinnöilisyys vaiko ympäristö?* [*Heredity or environment? (in criminology).*] Lakimies 1958, 4–5; p. 501–510.

Ilg, Frances: see Gesell, Arnold.

Katz, David: *Psykologian käsikirja.* [*Handbook of psychology.*] Ed. by Kai von Fieandt and Markus Sauri. Porvoo-Helsinki, Werner Söderström. XVII, 548 p. 1300,– FM.

Kokkonen, Olli: *Ateismista uskonnonhistoriallisena ongelmana.* [*On atheism as a problem of the history of religion.*] Teologinen aikakauskirja 1958, 5; p. 296–310.

Krohn, Sven: *Die normative Wertethik in ihrer Beziehung zur Erkenntnis und zur Idee der Menschheit.* Turku, University of Turku. 284 p. 1100,– FM. (=Annales Universitatis Turkuensis. Ser. B. Tom. 69.)

Kung-Fu-tse: *Keskustelut.* [*The Analects.*] Transl. by Toivo Koskikallio. Porvoo-Helsinki, Werner Söderström. 192 p. 400,– FM.

Laurikainen, K. V.: *Nykyaikaisen fysiikan todellisuuden käsitteestä.* [*On the concept of reality in modern physics.*] Suomalainen Suomi 1958, 1; p. 18–26.

Linkomies, Edwin: *Eino Kaila. Muistosanoja.* [*E. K. In memoriam.*] Valvoja 1958, 3; p. 93–94.

Littunen, Yrjö: *Nuoruusiän yksilöllistä ja sosiaalista kehitystä koskevia tutkimusongelmia.* [*Research problems of the individual and social development in adolescence.*] With English summary. Kasvatusopillinen aikakauskirja 1958, 4–5; p. 206–219.

Machiavelli, Niccolò: *Valtiollisia mietelmiä.* [*"Discorsi sopra la prima deca di Tito Livio."*] Transl. by Kaarlo af Heurlin. Porvoo-Helsinki, Werner Söderström. 370 p. 800,– FM.

Markkanen, Tonko: see Ahmavaara, Yrjö.

Matinolli, Eero: *Ihminen polttopisteessä.* [*Man in focus.*] (Review of: Urpo Harva: Ihminen etsii itseään.) Suomalainen Suomi 1958, 2; p. 91–96.

Nevanlinna, Rolf: *Tutkimus ja yhteiskunta.* [*Research and society.*] Suomalainen Suomi 1958, 4; p. 247–251.

Numelin, Ragnar: *Handstilens psykologi.* [*Psychology of handwriting.*] Nya Argus 1958, 14; p. 213–216.

Numelin, Ragnar: *Handstilens symbolik.* [*Symbolism of handwriting.*] Nya Argus 1958, 16; p. 243–246.

Oksala, Ohto: *Ruumiillinen ja henkinen työ.* [*Manual labour and intellectual work.*] Suomalainen Suomi 1958, 1; p. 9–14.

Palmgren, Pontus: *Ihmisen aivotoiminta ja hengenelämän synty.* [*The function of human brain and the origin of mental life.*] Valvoja 1958, 4; p. 152–161.

Pascal, Blaise: *Tankar I–II.* [*Pensées sur la religion.*] Transl. by Sven Stolpe. Helsinki, Söderström & Co. 173 & 174 p. 860,– FM.

Rantavaara, Irma: *Emerson, romanttinen idealisti.* [*E., a romantic idealist.*] (Review of: Ralph Waldo Emerson: Esseitä.) Suomalainen Suomi 1958, 7; p. 467–469.

Renvall, Pentti: *Totuuden valtakuntaa kohti. Abstraktisten yhteisöjen historiallisesta kehityksestä.* [*Towards the City of Truth. On the historical development of abstract communities.*] Suomalainen Suomi 1958, 7; p. 425–431.

Saarnio, Uuno: *Das System und die Darstellung der transfiniten Ordnungszahlen.* Helsinki, Gesellschaft für Logik und ihre Anwendungen. 208, 3 p. 2000,– FM.

Sainio, Matti A.: *Kasvatusopin tutkimus ja opetus Turun Akatemiassa.* [*Research and teaching of pedagogics in the Academy of Turku.*] Jyväskylä, Jyväskylän kasvatusopillinen Korkeakoulu. 194,

15 p. 500,– FM. (=Jyväskylän kasvatusopillisen Korkeakoulun julkaisuja XIV.)

Sorainen, Kalle: *Tanskan uskonnonfilosofiaa.* [*Danish philosophy of religion.*] Teologinen aikakauskirja 1958, 5; p. 287–295.

Tenkku, Jussi: *Onni ja varallisuus. Yhteiskunnallisten tieteiden asema Adam Smithin moraalifilosofiassa.* [*Happiness and wealth. The rôle of social sciences in A. Smith's moral philosophy.*] Suomalainen Suomi 1958, 1; p. 14–18.

Tenkku, Jussi: *Tietoteoreettinen monismi ja dualismi.* [*Epistemological monism and dualism.*] Teologinen aikakauskirja 1958, 3; p. 159–170.

Tuompo, Aarre: *Bemerkungen zur Rorschach-Methode.* Turku, University of Turku. 36 p. 125,– FM. (=Annales Universitatis Turkuensis. Ser. B. Tom. 58.)

Vahervuo, Toivo & Ahmavaara, Yrjö: *Johdatus faktorianalyysiin.* [*Introduction to factor analysis.*] Porvoo-Helsinki, Werner Söderström. 183 p. 950,– FM.

Voutilainen, Touko: *Kasvatuksen tarkoitus filosofisena ongelmana.* [*The purpose of education as a philosophical problem.*] With English summary. Kasvatusopilllinen aikakauskirja 1958, 2; p. 75–93.

Wright, Georg Henrik von: *Logiikka, filosofia ja kieli.* [*Logic, philosophy and language.*] Transl. by Jaakko Hintikka and Tauno Nykänen. Helsinki, Otava. 242, 1 p. 1000,– FM. (=Otavan filosofinen kirjasto 7.)

Wright, J. M. von: *An experimental study of human serial learning.* Helsinki, Societas Scientiarum Fennica, 1957. 161 p. 820,– FM. (=Commentationes Humanarum Litterarum XXIII, 1.)

Wright, J. M. von: *Siirtovaikutuksen ongelmasta.* [*On the problem of transfer effect.*] Jyväskylä, Jyväskylän kasvatusopillinen Korkeakoulu. 36 p. 200,– FM. (=Jyväskylän Kasvatusopillisen Korkeakoulun Psykologian laitoksen julkaisuja. N:o 21, 1958.)

NORWAY

(Antonius) Marcus Aurelius: *Til meg selv.* [*To myself.*] Transl. by Rebekka Hammering Bang. Dreyer 1957. Distr. 1958.

Aubert, Vilhelm & Messinger, Sheldon L.: *The Criminal and the Sick.* Inquiry (1958) I, 3; p. 137–160.

Beyer, Harald: *Nietzsche og Norden.* [*Nietzsche and Scandinavia.*] Bergen 1958. 204 pp. 15,– NCr. (=Årbok. Universitetet i Bergen. Hist.-ant. rekke Nr. 1, 1958.)

Broad, C. D.: *Philosophy (1) and (2).* Inquiry (1958) I, 2; p. 99–129.

Cavell, Stanley: *Must We mean What We Say?* Inquiry (1958) I, 3; p. 172–212.

Fenstad, Jens Erik: *Discussion of Rudolph Carnap: Introduction to Symbolic Logic and its Applications.* Inquiry (1958) I, 4; p. 254–258.

Föllesdal, Dagfinn: *Husserl und Frege. Ein Beitrag zur Beleuchtung der Entstehung der phänomenologischen Philosophie.* Aschehoug, Oslo 1958. 60 pp. 8,– NCr. (=Vid.-Akad. Skr. II. H.-F. Kl. 1958. No. 2.)

Föllesdal, Dagfinn: *Mates on Referential Opacity.* Inquiry (1958) I, 4; p. 232–238.

Galtung, Johan: *Notes on the Difference Between Physical and Social Sciences.* Inquiry (1958) I, 1; p. 7–34.

Gellner, Ernest: *Discussion of Werner Stark: The Sociology of Knowledge.* Inquiry (1958) I, 4; p. 247–253.

Gram Jensen, Sv.: *Determinisme og strafferet.* [*Determinism and criminal law.*] Tidskrift för Rettsvitenskap. (1958) 71, 3; p. 250–255.

Hellern, Victor: *Nykantianismens kulturfilosofi, særlig hos Ernst Cassirer.* [*The cultural philosophy of neokantianism, especially of Ernst Cassirer.*] Kirke og Kultur 1958, 3; p. 156–166.

Koppang, Ole: *Personalismen. En impuls i det 20. hundreårs åndsliv.* [*Personalism. An impulse in 20th century cultural life.*] Kirke og Kultur 1958, 9; p. 539–546.

Lasswell, Harold D.: *Clarifying Value Judgment: Principles of Content and Procedure.* Inquiry (1958) I, 2; p. 87–98.

Lönning, Per: *Tro og tanke efter Blaise Pascal.* [*Belief and thought according to Blaise Pascal.*] Land og Kirke, Oslo 1958. 307 pp. 24,– NCr.

Marcel, Gabriel: *Eksistens og erkjennelse. Betraktninger over det ontologiske mysterium.* [*"Position et approches concrètes du mystère ontologique."*] Transl. by Leif Tufte. Cappelen, Oslo 1958. 43 pp.

Mates, Benson: *On the Verification of Statements About Ordinary Language. Inquiry* (1958) I, 3; p. 161–171.

Mates, Benson: *Comments on Föllesdal.* Inquiry (1958) I, 4; p. 239–242.

Meløe, Jakob: *Dialogue on the Hypothetical Character of Logical Analysis.* Inquiry (1958) I, 1; p. 72–84.

Messinger, Sheldon L.: see Aubert, Vilhelm.

Nietzsche, Friedrich: *Hvorledes man blir hva man er.* [*How to become what you are.*] (From the preface to "Ecce homo") Horisont (1958) IV, 1; p. 30–33.

Næss, Arne: *Forskning og populærvitenskap.* [*Scientific research and popular science.*] Horisont (1958) IV, 6; p. 180–186.

Ofstad, Harald: *The Functions of Moral Philosophy.* Inquiry (1958) I, 1; p. 35–71. (Is also issued separate with some corrections at the Oslo University Press 1958. 45 pp. 3,50 NCr.)

Perelman, Charles: *The Theoretical Relations of Thought and Action.* Inquiry (1958) I, 2; p. 130–136.

Platons dialog ION. [*Plato's dialogue ION.*] Transl. with comm. by Egil A. Wyller. Aschehoug 1958; 61 pp. 6,– NCr. (=Klassiker-bibliotek, II.)

Prahl, Sören: *Kristen existentialisme.* [*Christian existentialism.*] Kirke og Kultur (1958) LXIII, 3; p. 133–146.

Rommetveit, Ragnar: *Epistemological Notes on Recent Studies of Social Perception.* Inquiry (1958) I, 4; p. 213–231.

Schöttz-Christensen, Aage: *Om naturerkendelsen i dansk digtning.* [*On apprehension of nature in Danish poetry.*] Edda (1958) LVIII, p. 214–235.

Skirbekk, Gunnar: *Nihilisme? Eit ungt menneskes forsök på å orientere seg.* [*Nihilism? A young man's attempt to orientate himself.*] Tanum 1958. 150 pp. 15,60/18. 90 NCr.

Skjervheim, Hans: *Deltakar og tilskodar.* [*Participator and spectator.*] (Mimeogr.) Oslo 1957. 17 pp. (=Inst. for sosiol. Stensilserie.)

Skjervheim, Hans: *Reason in Society and Modern Logic.* Inquiry (1958) I, 4; p. 243–246.

Stigen, Anfinn: *Marcus Aurelius og moderne filosofi.* [*Marcus Aurelius and modern philosophy.*] Samtiden (1958) LXVII, 9; p. 540–545.

Strahl, Ivar: *Om praktiska slutledningar inom juridiken.* [*On practical syllogisms in jurisprudence.*] Tidsskrift for Rettsvitenskap. (1958) 71, 1; p. 40–45.

Wasberg, Gunnar Christie: *Om forholdet mellom erkjennelse og verdianskuelse i historiefilosofien.* [*The Relationship between Perceptual Form and Assessment of Values in the Philosophy of History.*] With an English summary. Land og Kirke 1958. 113 pp. 8,– NCr.

SWEDEN

Ahlberg, Alf: *Bertrand Russel.* (In Swedish.) Studiekamraten XL, 4–5; p. 81–83.

Ahlberg, Alf: *Stat, rätt och barmhärtighet enligt katolsk socialfilosofi.* [*State, right and mercy according to catholic social philosophy.*] Samtid och Framtid (1958) XV, 4; p. 210–214.

Åkesson, Elof: *Religionens språk.* [*The language of religion.*] (Review of Jan T. Ramsey: Religious language. An empirical placing of theological phrases. London 1957.) Vår Lösen (1958) XLIX, 1; p. 30–34.

Aspelin, Gunnar: *Åke Petzäll. Minnesord.* [*Åke Petzäll in memoriam.*] Kungl. Humanistiska vetenskapssamfundet i Lund. Årsberättelse 1957/58; p. 1–15.

Aspelin, Gunnar: *Tankens vägar. En översikt av filosofiens utveckling. 1–2.* [*The ways of thought. A survey of the development of philosophy. 1–2.*] Almqvist & Wiksell, Geber, Stockholm 1958. Vol. 1. 256 p. 24,– SwCr. Vol. 2. 350 p. 30,– SwCr.

Aspelin, Kurt: *Thomas Thorild – upplysningsman och folktribun.* [*Thomas Thorild – philosopher of enlightenment and tribune of the people.* Vår tid (1958) XIV, 6; p. 211–226.

Asplund, Rudolf: *Inför filosofen och diktaren Hedenius.* [*Face to face to the philosopher and poet Hedenius.*] (Review of Ingemar Hedenius: I stället för samtal. (Instead of conversations.)) Vår lösen (1958) XLIX, 9; p. 293–295.

Berg, Jan: *A note on reduction sentences.* Theoria (1958) XXIV, 1; p. 1–8.

Bergmann, Gustav: *Analyticity.* Theoria (1958) XXIV, 2; p. 71–93.

(Birgitta) *Den heliga Birgitta. Himmelska uppenbarelser. 1–3.* [*Revelations, 1–3.*] Transl. by Tryggve Lundén. Allhem, Malmö. Vol. 1. 346 p. 1957. 40,– SwCr. Vol. 2. 320 p. 1958. 40,– SwCr. Vol. 3. 355 p. 1958. 40,– SwCr.

Black, Max: *Notes on the meaning of 'rule'.* I–II. Theoria (1958) XXIV, 2; p. 107–126 & XXIV, 3; p. 139–161.

Brunius, Teddy: *Andreas Rydelius och hans filosofi.* [*Andreas Rydelius and his philosophy.*] Insikt och Handling (1958) II; p. 5–21.

Canning, Wather: *"Den moderna människan sjunger inte". Tankar och åsikter av Bertrand Russell, upptecknade av den dansk-amerikanske journalisten Wather Canning under hans besök hos den engelske filosofen.* ["*Modern man does not sing.*" *Thoughts and opinions of Bertrand Russell. Notes taken by the Danish-American journalist Wather Canning during his visit to the English philosopher.* Värld och vetande (1958) VIII, 12; p. 367–370.

Carney, Edward: *Voltaire's Candide and the English reader. Notoriety and popularity.* Moderna språk (1958) LII, 3; p. 234–251.

Collinder, Björn: *Språkforskningens plats i vetenskapernas republik.* [*The place of linguistics in the republic of sciences.*] Kungl. Vetenskapssocieteten i Upsala. Årsbok 1958, p. 55–69.

Dahm, Erland: *Björn Sjövalls teorier.* [*Björn Sjöval's theories.*] Kulturkontakt 1958; p. 18–24.

Düring, Ingemar: *Aristotle in the ancient biographical tradition.* Almqvist & Wiksell, Stockholm 1957. 490 p. 32,– SwCr. (=Göteborgs universitets årsskrift, vol. 63: 2 & =Studia Graeca et Latina Gothoburgensia. 5.)

Eeg-Olofsson, Ansgar: *Även den religiösa människan har problem.* [*The religious man also has problems.*] Kristet Forum 1958, 6; p. 131–134.

Eklund, Harald: *De teologiska fakulteternas framtid.* [*The future of the faculties of theology.*] Samtid och framtid (1958) XV, 2; p. 95–98.

Eklund, Harald: *Om "tror-satser".* [*On "believe-sentences".*] Svensk Teologisk Kvartalskrift (1958) XXXIV; p. 150–152.

Eklund, Harald: *Personligt och sakligt i religionsfrågor.* [*Thoughts and facts on religious matters.*] Natur och Kultur, Stockholm 1958. 174 p. 14,50 SwCr.

Eklund, Harald: *Religionen skapar problem. En orientering i religionsfilosofi.* [*Religion creates problems. An orientation in the philosophy of religion.*] Svenska bokförlaget (Norstedt), Stockholm 1958. 64 p. 6,50 SwCr. (=Scandinavian university books.)

Eklund, Harald: see Hedenius, Ingemar.

Ekman, Rolf: Platons estetik. [*The aesthetics of Plato.*] Insikt och Handling (1958) II; p. 22–32.

Ellegård, Alvar: *Darwin and the general reader. The reception of Darwin's theory of evolution in the British periodical press, 1859–1872.* Almqvist & Wiksell, Stockholm 1958. 393 p. 28,– SwCr. (=Göteborgs universitets årsskrift, vol. LXIV, 7; 1958.)

En Goethe-bok. Till Algot Werin. [*Goethe essays. In honour of Algot Werin.*] Gleerup, Lund, 1958. 341 p. 25,– SwCr.
From the contents:

Åkesson, Elof: *Vetenskap – konst – religion. En kort anteckning till ett stort tema hos Goethe.* [*Science – art – religion. A short note about a large theme in Goethe.*] p. 33–39.

Valdén, Nils Gösta: *Kring Goethe, Nietzsche och Vilhelm Ekelund.* [*About Goethe, Nietzsche and Vilhelm Ekelund.*] p. 323–335.

Landquist, John: *Geijer och Goethe.* [*Geijer and Goethe.*] p. 220–236.

Fehrman, Carl: *Om Friedrich Schillers Traktat über naive und sentimentalische Dichtung.* [*On Friedrich Schiller's Traktat über naive und sentimentalische Dichtung.*] Insikt och Handling (1958) II; p. 33–46.

Fogelklou, Emilia: *Form och strålning. Åskådningsfragment.* [*Form and radiation. A fragment of a philosophy of life.*] Bonnier, Stockholm 1958. 246 p. 19,50 SwCr.

Fredén, Gustaf: *Orpheus and the goddess of nature.* Almqvist & Wiksell, Stockholm 1958. 179 p. 15,– SwCr. (=Göteborgs universitets årsskrift, vol. LXIV, 6; 1958.)

Frykholm, Lars: *Till Montesquieus maktdelningslära.* [*Montesquieu's theory of the distribution of power.*] Statsvetenskaplig Tidskrift (1958) LXI, 2–3; p. 203–205.

Furberg, Mats & Nordenstam, Tore: *Om "tror-satser".* [*On*

"believe-sentences."] Svensk Teologisk Kvartalskrift (1958) XXXIV; p. 146–150.

Götlind, Erik: *Three theories of emotion. Some views on philosophical method.* Gleerup, Lund (& Munksgaard, Copenhagen) 1958. 150 p. 16,– SwCr. (=Library of Theoria, 3.)

Granlund, Sten: *Den nya syntesen.* [*The new synthesis.*] Litteraturförlaget, Stockholm 1958. 103 p. 6,50 SwCr. Preface by Alf Ahlberg.

Grundström, Birger: *Heisenbergs osäkerhetsrelation i gymnasieundervisningen.* [*Heisenberg's uncertainty relation in secondary school teaching.*] Elementa (1958) 41, 2; p. 105–113.

Guitton, Jean: *Konsten att tänka och handla.* [*"Apprendre à vivre et à penser".*] Transl. by Vilma Roos. Postscript by Daniel Andreae. Natur och Kultur, Stockholm 1958. 102 p. 8,75 SwCr.

Gustafsson, Berndt: *Kierkegaard och kyrkoåret.* [*Kierkegaard and the ecclesiastical year.*] Svensk Teologisk Kvartalskrift (1958) XXXIV; p. 96–110.

Halleman, Gunnar (pseud. for Carlson, Gunnar): *Människan – världsalltet.* [*Man and the universe.*] (Seelig), Stockholm 1957. 87 p. 4,75 SwCr.

Hansen, J. Schmidt: *Rättsstatens fader. Forskaren och människan Severin Christensen.* [*The father of the legal state. Severin Christensen, the scientist and the man.*] Samtid och Framtid (1958) XV, 1; p. 10–14.

Hansen, Valdemar: *Om Niels Bohrs filosofi.* [*The philosophy of Niels Bohr.*] Nordisk Tidskrift (1958) XXXIV, 5; p. 333–335.

Hartnack, Justus: *Dreams.* Theoria (1958) XXIV, 3; p. 162–171.

Hartnack, Justus: *Filosofiska problemställningar.* [*Philosophical problems and philosophical arguments.*] Transl. by Alarik Roos. Svenska Bokförlaget (Norstedt), Stockholm 1958. 256 p. 14,50 SwCr.

Hedenius, Ingemar: *Årstider.* [*Seasons.*] Kulturkontakt 1958; p. 24–25.

Hedenius, Ingemar: *I stället för samtal.* [*Instead of conversations.*] Bonniers, Stockholm 1958. 122 p. 11,50 SwCr.

Hedenius, Ingemar: *Tankar om religionen.* [*Reflections on religion.*] Kulturkontakt 1958; p. 17–19.

Hedenius, Ingemar: *Tro och livsåskådning.* [*Belief and philosophy of life.*] Bonnier, Stockholm 1958. 295 p. 9,50 SwCr. (=Aldusböckerna.)

Hedenius, Ingemar & Eklund, Harald: *Om teologien och kyrkan.* [*On theology and the church.*] Lindblad, Uppsala 1958. 96 p. 4,25 SwCr. (=Kristen debatt 3.)

Hegeland, Hugo: *Protestantism och kapitalism.* [*Protestantism and capitalism.*] Samtid och Framtid (1958) XV, 1; p. 52–56.

Hejll, Richard: *Kulturbegreppets upplösning.* [*Dissolution of the culture concept.*] Den Enskilde 1958, 1; p. 1–8.

Hejll, Richard: *Vad betyder satsen: Subjektiviteten är sanningen? Ett försök till utredning.* [*What does the sentence – "subjectiveness is truth" – mean? A temptative investigation.*] Den Enskilde 1958, 2 & 3; p. 17–22 & 29–33.

Hellner, Jan: *Legal philosophy in the analysis of tort problems.* (Separate from Scandinavian studies in law, vol. 2; p. 149–176.) Almqvist & Wiksell, Stockholm 1958. (=Studia juridica Stockholmiensia 4.)

Hemrin, Sven: *Religionsfilosofi – vad är det?* [*Philosophy of religion – what is it?*] Tro och liv 1958, 5; p. 199–206.

Hesselgren, Sven: *Konstupplevelsens psykologi.* [*The psychology of experiencing art.*] Vår lösen (1958) XLIX, 4; p. 116–125.

Hillerdal, Gunnar: *Religionsfilosofien och kristendomens sanningsfråga.* [*Philosophy of religion and the problem of truth in Christianity.*] Insikt och Handling (1958) II; p. 47–62.

Hillerdal, Gunnar: *Teologisk och filosofisk etik. Brytningar och synteser i etikens historia från antiken till nutiden.* [*Theological and philosophical ethics. Controversies and syntheses in the history of ethics from antiquity to our days.*] Svenska Bokförlaget (Norstedt), Stockholm 1958. 273 p. 26,– SwCr. (=Scandinavian university books.)

Hiorth, Finngeir: *On defining "Word".* Studia Linguistica (1958) XII, 1; p. 1–26.

Holte, Ragnar: *Beatitudo och sapientia. Augustinus och de antika filosofskolornas diskussion om människans livsmål.* [*Beatitudo and spaientia. Augustine and the discussion of man's telos in the philosophical schools of antiquity.*] With a summary in German. Svenska Kyrkans Diakonistyrelses bokförlag, Stockholm 1958. 416 p. 30,– SwCr.

Holte, Ragnar: *Res et verba. Slutreplik till docent Nygren.* [*Res et verba. Final reply to Dr. Nygren.*] Svensk Teologisk Kvartalskrift (1958) XXXIV; p. 69–77.

Hook, Sidney & Russell, Bertrand: *Kapitulation och ödeläggelse. Tre debattinlägg.* [*Capitulation and ruin. Three debate contributions.*] Kulturkontakt 1958; p. 3–11.

Jaspers, Karl: *Sanning, frihet och fred.* [*Truth, freedom and peace.*] Samtid och framtid (1958) XV, 4; p. 196–202.

Johannesson, Rudolf: *Att tänka logiskt.* [*On thinking logically.*] Ehlin, Stockholm 1958. 76 p. 3,60 SwCr.

Johannesson, Rudolf: *Logikundervisningen på gymnasiet.* [*The teaching of logic in secondary schools.*] Pedagogisk Debatt (1958) VI, 1; p. 11–13.

Jonson, Erik: *Värdegåtan.* [*The riddle of value.*] Fönstret (1958) 35, 2–4; p. 32–33, 62, 83.

Jonsson, Inge: *Emanuel Swedenborg och Yttersta Domen. Ett litterärt 200-årsminne.* [*Emanuel Swedenborg and the last judgment. A twohundred year anniversary.*] Ord och bild (1958) 6; p. 417–428.

Kalén, Nils: *Gyllene visdom. Läsning för sökande själar.* [*Golden wisdom. Reading for searching souls.*] Hälsan, Linköping 1958. 174 p. 9,50 SwCr. (=Vidare vyer. 2.)

Kanger, Stig: *New foundations for ethical theory.* Author, Uppsala. (Mimeographed.) 1957. Part 1. 42 p. 10,– SwCr.

Khatchadourian, Haig: *What is rationality?* Theoria (1958) XXIV, 3; p. 172–187.

Kruse, Fr. Vinding: *Berkeley and the esse est percipi-principle.* Theoria (1958) XXIV, 2; p. 127–130.

Landberg, Georg: *Historietolkning.* [*Interpretation of history.*] Historisk Tidskrift (1958) Ser. II, Vol. XXI, 4; p. 108–121.

Lång, Helmer: *Gossekärlek och platonism.* [*Pederasty and Platonism.*] Värld och vetande (1958) VIII, 4; p. 107–110.

Lange, Birgit: *Försonaren.* [*The atoner.*] Gleerup, Lund 1957. 178 p. 12,50 SwCr.

Lange, Birgit: *Hur kan förtröstan leva?* [*How can trust live?*] Gleerup, Lund 1957. 79 p. 7,50 SwCr.

Lange, Birgit: *Vårt behov av tröst.* [*Our need of consolation.*] Kristet Forum 1958, 9–10; p. 197–198.

Langer, Susanne K.: *Filosofi i en ny tonart. En undersökning av tänkandets, ritens och konstens symboler.* [*"Philosophy in a new key. A study in the symbolism of reason, rite and art."*] Transl. by Elof Åkesson. Almqvist & Wiksell/Geber, Stockholm 1958. 363 p. 19,50 SwCr. (=Levande debatt.)

Larsson, Hans: *Studier och meditationer.* [*Studies and meditations.*] New edition. Bonnier, Stockholm 1958. 136 p. 5,50 SwCr. (=Bonnierbiblioteket.)

Leander, Folke: *Censorsförhör i filosofiska och andra orienteringsämnen.* [*The examination by the public delegates in philosophical and other orientation subjects.*] Pedagogisk Debatt (1958) VI; p. 80–83.

Levander, Hans: *Emilia Fogelklou.* (In Swedish.) Studiekamraten XL, 11–12; p. 225–228.

Lindblom, Joh.: *Heliga Birgittas uppenbarelser på nutidssvenska. Tillika en religionspsykologisk inledning till deras studium.* [*St. Birgitta's revelations in modern Swedish. An introduction from the point*

of view of religious psychology.] Svensk Teologisk Kvartalskrift (1958) XXXIV; p. 213–224.

Lindblom, Johannes: *Till frågan om den religiösa mystikens väsen.* [*On the problem of the nature of religious mysticism.*] Svensk Teologisk Kvartalskrift (1958) XXXIV; p. 1–15.

Lindroth, Sten: *Den moderna naturvetenskapens födelse.* [*The birth of modern science.*] Studiekamraten (1958) XL, 10; p. 219–224.

Lindroth, Sten: *Naturvetenskaperna och kulturkampen under frihetstiden.* [*Science and enlightenment in Sweden 1740–1765.*] With English summary. Lychnos 1957–58; p. 181–193.

Löb, Hermann: Martin Buber. (In Swedish.) Judisk Tidskrift (1958) XXXI; p. 11–14.

Lönnebo, Martin: *Harald Eklunds religionsfilosofi.* [*Harald Eklund's philosophy of religion.*] Årsbok för Kristen Humanism (1958) XX; p. 44–48.

Löwegren, Gunnar: *François Quesnay – läkare och samhällsfilosof.* [*François Quesnay – physician and social philosopher.*] Samtid och Framtid (1958) XV, 4; p. 225–227.

Lundgren, Gustaf: *Anders Nygren och psykologien.* [*Anders Nygren and psychology.*] Svensk Teologisk Kartalskrift (1958) XXXIV; p. 281–298.

Machiavelli, Niccolò: *Fursten.* [*"Il principe".*] Transl. by Karin Hybinette. With postscript by Erik Lönnroth. Natur och Kultur, Stockholm 1958. 152 p. 7,75 SwCr. (=Levande Litteratur.)

Marcus Aurelius: *Självbetraktelser.* [*Meditations.*] Transl. by Ellen Wester. New ed. Forum, Stockholm 1958. 192 p. 6,50 SwCr. (=Forumbiblioteket. 40.)

Marc-Wogau, Konrad: *The argument from illusion and Berkeley's idealism.* Theoria (1958) XXIV, 2; p. 94–106.

Moritz, Manfred: *Åke Petzäll.* (In Swedish.) Vetenskaps-Societeten i Lund. Årsbok 1957; p. 117–123.

Nielsen, Kai: *Good reasons in ethics: An examination of the Toulmin-Hare controversy.* Theoria (1958) XXIV, 1; p. 9–28.

Nordenstam, Tore: see Furberg, Mats.

Nordström, John: *Till Cartesius' ikonografi.* [*On the iconography of Descartes.*] With English summary. Lychnos 1957–58; p. 194–250.

Norinder, Yngve: *Om Charles Darwin och psykologien.* [*About Charles Darwin and psychology.*] Skola och Samhälle (1958) XXXIX, 6–7 p. 165–175.

Nygren, Gotthard: *Res contra verba? Slutreplik till licentiat Holte.* [*Res contra verba? Final reply to Mr. Holte.*] Svensk Teologisk Kvartalskrift (1958) XXIV; p. 77–80.

Nyman, Alf: *Begreppet lyrisk erfarenhet. Kunskapsteoretiska och*

estetiskt-psykologiska studier i symbolistisk och realistisk diktning. [*The concept of lyrical experience. Epistemological and æsthetic-psychological studies in symbolistic and realistic poetry.*] With summary in French. Gleerup, Lund 1958. 423 p. 40,– SwCr. (=Skrifter utgivna av Kungl. Humanistiska Vetenskapssamfundet i Lund, 55.)

Petander, Karl: *Historien och personlighetstanken.* [*History and the idea of personality.*] Kooperativa förbundets bokförlag, (distr.) Stockholm 1957. 260 p. 22,– SwCr.

Pettersson, Olof: *Magic – religion. Some marginal notes to an old problem.* Ethnos (1957) XXII; p. 109–119.

Petzäll†, Åke: *Ironin som filosofiskt instrument.* [*Irony as a philosophical instrument.*] Insikt och Handling (1958) II; p. 63–76.

Petzäll†, Åke: *Den kristna etikens dialektik såsom historiskt fenomen.* [*The dialectics of Christian ethics as a historical phenomenon.*] Svensk Teologisk Kvartalskrift (1958) XXXIV; p. 299–313.

(Physiologus) *En bok om naturens ting.* [*A book on natural things.*] Transl. by Brita-Stina Nordin-Pettersson from a German translation by K. Ahrens: *Das Buch der Naturgegenstände* 1892. Sällskapet Bokvännerna, Stockholm 1957. 85 p. (No price stated.)

Raulin, A.: *Religiositet och moralisk förpliktelse.* [*Religiosity and moral obligation.*] Credo (1958) XXXIX, 1; p. 3–12.

Regnéll, Hans: *Semantik. Filosofiska och språkvetenskapliga grundfrågor inom betydelseläran.* [*Semantics. Basic philosophical and linguistic problems within the study of meaning.*] Svenska Bokförlaget (Norstedt), Stockholm 1958. 187 p. 14,– SwCr. (=Scandinavian university books.)

Roos, Alarik: *Kaosmänniskan och hennes bekämpande.* [*The chaotic personality and how to combat him.*] Samtid och Framtid (1958) XV, 4; p. 247–251.

Rosén, Sven: *Kan man reda sig utan filosofi?* [*Is it possible to live without philosophy?*] Värld och Vetande (1958) VIII, 1; p. 21–24.

Rosén, Sven: *Vetenskap och religion.* [*Science and religion.*] Värld och Vetande (1958) VIII, 4; p. 118–126.

Rüdiger, Helmut: *Misstag kring Marx.* [*Missapprehensions about Marx.*] Kulturkontakt 1958; p. 38–42.

Ruin, Hans: *Drömmen om den rena poesien.* [*The dream about pure poetry.*] Ord och Bild (1958) 6; p. 417–428.

Russell, Bertrand: *Filosofins problem.* [*"The problems of philosophy".*] Transl. by Anders Byttner. Postscript by Konrad Marc-Wogau. Natur och Kultur, Stockholm 1958. 161 p. 9,25 SwCr. (=Levande litteratur.)

Russell, Bertrand: *Varför jag inte är kristen och andra essäer om religion och besläktade ämnen.* [*"Why I am not a Christian and other essays on religion and related subjects."*] Ed. with an appendix

by Paul Edwards. Transl. by Anders Byttner. Natur och Kultur, Stockholm 1958. 252 p. 19,50 SwCr.

Russell, Bertrand: see Hook, Sidney.

Ryding, Erik: *The distinction between one-one relation and one-many relation.* Theoria (1958) XXIV, 1; p. 48–51.

Ryding, Erik: *The sense of "Smoking permitted".* Theoria (1958) XXIV, 3; p. 188–190.

Saarnio, Uuno: *Über die Gesamtheit und ihre Einteilung.* Theoria (1958) XXIV, 1; p. 29–38.

Sandblad, Henrik: *Nils Celsius i Livland och Göteborg. En episod i karolinsk lärdomshistoria.* [*Nils Celsius in Livonia and Gothenburg.*] With English summary. Lychnos 1957–58; p. 63–84.

Simonsson, Tord: *Face to face with Darwinism. A critical analysis of the Christian front in Swedish discussion of the later nineteenth century.* Gleerup, Lund 1958. 192 p. 16,50 SwCr.

Sjöbring, Henrik: *Struktur och utveckling. En personlighetsteori.* [*Structure and development. A theory of personality.*] Edited by Erik Essen-Möller & Carl-Erik Uddenberg. Gleerup, Lund 1958. 246 p. 20,– SwCr.

Sørensen, Helge: *Tro og Viden.* [*Faith and knowledge.*] Religion och Kultur (1958) XXIX, 2; p. 43–48.

Stjernquist, Per: *Gränsområden mellan rättsvetenskap och samhällsvetenskap.* [*Common fields of jurisprudence and the social sciences.*] Statsvetenskaplig Tidskrift (1958) LXI, 2–3, p. 141–150.

Törnebohm, Håkan: *Fysik och filosofi. Ett vetenskapsteoretiskt studium. Appendix: On explanations, predictions, and theory in physics. A case study.* [*Physics and philosophy. Studies in the theory of science.*] Almqvist & Wiksell, Stockholm 1957. 141 p. 15 SwCr. (=Göteborgs universitets årsskrift. (1957) Vol. 63: 4.)

Törnebohm, Håkan: *Notes on modal operators.* Theoria (1958) XXIV, 2; p. 130–135.

Törnebohm, Håkan: *Om förklaringar i historien.* [*On explanations in history.*] Insikt och Handling (1958) II; p. 77–85.

Törnebohm, Håkan: *Outlines of a Boolean tensor algebra with applications to a lower functional calculus.* Theoria (1958) XXIV, 1; p. 39–47.

Tranøy, Erik: *Reply to Erik Ryding.* Theoria (1958) XXIV, 3; p. 190–191.

Utterström, Gustaf: *Socialhistoria och socialhistorisk metod. Ett genmäle mot Börje Hanssen.* [*Social history and the method of historical sociology. A rejoinder to Börje Hanssen.*] Historisk Tidskrift (1958) Ser. II, Vol. XXI, 1; p. 68–87.

Världsreligionernas kärnord. [*The pithy words of the*

world religions.] New ed. by Ernst Arbman. Svenska Bokförlaget (Bonnier), Stockholm 1957. 136 p. 8,50 SwCr.

With contributions by Tor Andrae, Bernhard Karlgren, Ernst Arbman, Efraim Briem, H. S. Nyberg, Gunnar Rudberg, Gösta Lindeskog.

Vision och gestalt. Studier tillägnade Ragnar Josephson. [*Vision and form. In honour of Ragnar Josephson.*] Edited by Mårten Liljegren and Sven Sandström. Natur och Kultur, Stockholm 1958. 361 p. 34,– SwCr.

From the contents:

Holmberg, Olle: *Tre streck på papperet.* [*Three strokes on the paper.*] p. 328–340.

Wedberg, Anders: *Filosofiens historia. 1. Antiken och medeltiden.* [*The history of philosophy. 1. Antiquity and the middle ages.*] Bonnier, Stockholm 1958. 196 p. 18,50 SwCr.

Wifstrand, Albert: *Den gyllene kedjan.* [*The golden chain.*] With English summary. Lychnos 1957–58; p. 1–13.

Wigforss, Ernst: *Om provisoriska utopier.* [*On provisional utopias.*] Insikt och Handling (1958) II; p. 86–119.

Wood, Alan: *Bertrand Russell, den passionerade skeptikern.* [*"Bertrand Russell. The passionate sceptic."*] Transl. by Anders Byttner. Natur och Kultur, Stockholm 1958. 268 p. 19,50 SwCr.

Wright, G. H. von: *Eino Kaila †.* (In English.) Theoria (1958) XXIV, 3; p. 137–138.

BOOKS RECEIVED

Apostel, Léo & Jonckherre, A.R. & Matalon, Benjamin: *Logique, apprentissage et probabilité.* Presses Universitaires de France, Paris 1959. 186 p. F 900,– (=Études d'épistémologie génétique VIII).

Apostel, L. & Mandelbrot, B. & Morf, A.: *Logique, language et théorie de l'information.* Presses Universitaires de France, Paris 1957. 207 p. F. 860.

Apostel, L. & Mays, W. & Morf, A. & Piaget, J.: *Les liaisons analytiques et synthétiques dans les comportements du sujet.* Presses Universitaires de France, Paris 1957. 148 p. F. 600,–.

Chisholm, Roderick M.: *Perceiving: A philosophical study.* Cornell University Press, Ithaca N. Y. (1957). 203 p. $ 2,75.

Contemporary Philosophical Problems. Selected readings. Ed. by Y. H. Krikorian and A. Edel. Macmillan Company, New York (1959). 712 p. $ 7,–.

Ethics and other knowledge. The Catholic University of America, Washington. 236 p. (=Proceedings of The American Catholic Philosophical Association. Vol. XXXI.)

Fang, Thomé H.: *The Chinese view of life. The philosophy of comprehensive harmony.* The Union Press, Hong Kong (1957). 274 p.

Gormsen, Marius: *Tilværelse I.* With English summary. (D.B.K.) Copenhagen 1957. 224 p.

Gréco, P. & Piaget, J.: *Apprentissage et connaissance.* Presses Universitaires de France, Paris 1959. 185 p. F. 900,–. (=Études d'épistémologie génétique VII.)

Gurwitsch, Aron: *Théorie du champ de la conscience.* Desclée de Brouwer, Bruges (1957). 350 p. Fr.B. 195. (=Textes et études anthropologiques.)

Hallett, H. F.: *Benedict de Spinoza. The elements of his philosophy.* University of London-The Athlone Press, London 1957. 171 p. 25/–.

Hegge, Hjalmar: *Erkjennelse og virklighet. Et bidrag til kritikk av teorien om sansekvalitetenes subjektivitet.* Mit deutscher Zusammenfassung: Erkennen und Wirklichkeit. Universitetsforlaget, Oslo 1957. 170 p. NCr. 9,–.

Holland, Virginia L.: *Counterpoint: Kenneth Burke and Aristotle's Theories of Rhetoric.* Philosophical Library, New York 1959. 128 p. $ 3,75.

Hwang-Tsong: *The last principle.* (64 Tah-Tung Road, Chung-Lih, Taiwan, China.) 1958. 70 p.

Il pensiero americano contemporano. Filosofia-Epistemologia-Logica. Direzione dell'opera: F. Rossi-Landi. Collaboratori: F. Barone, L. Borghi, G. Dorfles, P. Rossi, F. Rossi-Landi, U. Scarparelli, V. Somenzi, G. Vaccarino, A. Visalberghi. Edizioni di Comunità, Milano 1958. 341 p.

Il pensiero americani contemporano. Scienze sociali. Direzione dell'opera: F. Rossi-Landi. Collaboratori: F. Barbano, M. Calamandrei, R. Canestrari e A. Marzi, A. Giuliani, B. Griziotti, J. Griziotti Kretschmann, L. Saffirio, G. Sartori. Edizioni de Comunità, Milano 1958. 391 p.

Jacoby, J.E.: *Across the Night. Adventures in the Supranormal.* Philosophical Library, New York 1958. 110 p. $ 3,75.

Johnson, Martin: *Science and the meanings of truth.* Studies introductory to asking what is meant today by physical explanation of Nature, by mechanisms of cause and effect, and by a claim that scientific knowledge is true. Farber & Farber, London 1956. 179 p. 12/6.

Johnstone, Henry W. Jr.: *Philosophy and argument.* The Pennsylvania State University Press 1959. 141 p. $ 4,–.

Katzenellenbogen, Solomon: *Analyzing psychotherapy.* Philosophical Library, New York (1958). 126 p. $ 3,–.

Koebner, Richard & Gertrud: *Vom Schönen und seiner Wahrheit. Eine Analyse ästhetischer Erlebnisse.* Walter de Gruyter, Berlin 1957. 126 p. DM 12,–.

Koestler, Arthur: *The Sleepwalkers. A history of man's changing vision of the Universe.* With an Introduction by Herbert Butterfield. Hutchinson & Co Ltd, London 1959. 624 p. 25,–.

Krohn, Sven: *Die normative Wertethik in ihrer Beziehung zur Erkenntnis und zur Idee der Menschheit.* Turun Yliopiston Kustantama, Turku 1958. 284 p. FM 1100,–. (=Turun Yliopiston Julkaisuja Ser. B. Tom. 69.)

La philosophie de l'histoire de la philosophie. Articles de E. Castelli, A. Dempf, M. de Corte, A. del Noce, E. Garin, O. Gigon, H. Gouhier, M. Gueroult, E. Husserl, E. Lombardi, P. Valori, A. Wagner. Istituto di Studi Filosofici, Roma & Librairie Philosophique J. Vrin, Paris 1956. 208 p. (=Bibliothèque d'histoire de la philosophie.)

Lamouche, André: *La théorie harmonique, III: Psychologie.* Gauthier-Villars et Cie, Paris (1957). 470 p. F 1400,–.

Loehrich, Rolf R.: *Modus operandi. The methods of philosophical engagement.* The Compass Press, McHenry, Illinois. (1956) 117 p. $ 4,75.

Logical positivism. Edited by A. J. Ayer. The Free Press, Glencoe, Illinois, 1959. 455 p.

Lomholt, Esbern: *Home conditions. A sociomedical study of 1066 hospitalized patients with skin and veneral diseases.* Rosenkilde and Bagger, Copenhagen (1958). 100 p. DCr 14,–.

Mackinnon, D. M.: *A study in ethical theory.* Adam & Charles Black, London (1957). 280 p. 20/–.

Masor, Nathan: *The new psychiatry.* Philosophical Library, New York 1959. 155 p. 3,75 $.

Mercier, André: *Thought and being. An inquiry into the nature of knowledge.* Final text rendered by Alan Bloch. Verlag für Recht und Gesellschaft Basel 1959. (=Studia Philosophica Supplementum 9.) 156 p. Fr. 18,50.

Merton, Robert: *Social Theory and social structure.* Revised and enlarged edition. The Free Press, Glencoe, Illinois. 645 p. $ 7,50.

Moloney, James Clark: *Fear: contagion and conquest.* Philosophical Library, New York (1957). 140 p. $ 3,75.

Mudry, Joseph: *Philosophy of atomic physics.* Philosopical Library, New York 1958. 136 p. $ 3,75.

Nyman, Alf: *Begreppet lyrisk erfarenhet. Kunskapsteoretiska och estetiskt-psykologiska studier i symbolistisk och realistisk diktning.* C. W. K. Gleerup, Lund 1958. 423 p. SwCr 40,–. (=Skrifter utgivna av Kungl. Humanistiska Vetenskapssamfundet i Lund. LV.)

Popper, Karl R.: *The Logic of scientific discovery.* Hutchinson, London 1959. 480 p. 50/–.

Prior, A. N.: *Time and modality.* Being the John Locke lectures for 1955–6 delivered in the University of Oxford. Clarendon Press: Oxford Press 1957. 148 p. 25/–.

Psychoanalysis scientific method and philosophy. A synposium edited by Sidney Hook. Contributions by Heinz Hartmann, Lawrence S. Kubie, Ernest Nagel, Abram Kardiner, Michael Scriven, Philipp Frank and many others. New York University Press, New York 1959. 370 p. $ 5,–.

Rothstein, Jerome: *Communication, organization and science.* With a foreword by C. A. Muses. The Falcon's Wing Press, Indian Hills (1958). 110 p. $ 3,50.

Runes, Dagobert D.: *A Dictionary of Thought. From my writings and from my evenings.* Philosophical Library, New York 1959. 152 p. $ 5,–.

Runes, Dagobert D.: *Pictorial History of Philosophy.* Philosophical Library, New York 1959. 406 p. $ 15,–.

Ryding, Erik: *Den svenska filosofiens historia.* Natur och Kultur, Stockholm 1959. 169 p. SwCr. 11,50.

Saarnio, Uuno: *Logikens uppkomst och väsen.* Övers. från finskan av Jouko Ketonen. Sällskapet för logik och dess tillämpningar, Helsingfors (1957). 30 p. (I distribution hos Seelig & Co., Stockholm. SwCr 7,–.)

Schachter, Stanley: *The psychology of affiliation. Experimental studies of the sources of gregariousness.* Stanford University Press, Stanford, Calif. 1959. 141 p. (=Stanford Studies in Psychology I.) $ 3,75.

Schiavone, Michele: *Problemi filosofici in Marsilio Ficino.* Marzorati-Editore, Milano (1957). 328 p. L. 2500 (=Pubblicazioni dell' Istituto di Filosofia dell'Università di Genova VI.)

Schopenhauer, Arthur: *The world as will and representation.* Translated by E. F. J. Payne. 1–2. The Falcon's Wing Press, Indian Hills, Colorado. 534 & 687 p. $ 17,50.

Schultzer, Bent: *Relativitet i filosofisk belysning.* Gads forlag, Copenhagen 1957. 344 p.

Söderling, Bertil: *Utveckling och personlighetsbildning i tidiga barndomsår.* AB Fack- och Specialböcker, Stockholm (1957). 189 p. SwCr 15,–.

Stahl, Gerold: *Enfoque moderno de la logica clasica.* Ediciones de la universidad de Chile 1958. 187 p.

Suppes, Patrick: *Introduction to logic.* D. van Nostrand Company, Inc., Princeton, New Jersey 1957. 312 p. $ 5,50. (=The university series in undergraduate mathematics.)

Tallet, Jorge: *The absolute being.* Philosophical Library, New York 1958. 74 p. $ 3,–.

Theophilus, Johannes: *The book counsellor or proverbs of living wisdom.* The Netherlands, 1959. 172 p.

Wedberg, Anders: *Filosofiens historia. Nyare tiden till romantiken.* Bonniers, Stockholm, 1959. 184 p. SwCr. 18,50.

Weizsaecker, Viktor v.: *Le cycle de la structure (Der Gestaltkreis).* Trad. par M. Foucault & D. Rocher. Préface du Dr H. Ey. Desclée de Brouwer, Bruges (1958). F 195,–. (=Bibliothèque neuro-psychiatrique de langue française.)

Vetter, George B.: *Magic and religion. Their psychological nature, origin and function.* Philosophical Library, New York 1958. 555 p. $ 6,–.

Wilson, N. L.: *The concept of language.* University of Toronto Press, 1959. 153 p. $ 4,95.

MIND

A Quarterly Review of Psychology and Philosophy

OCTOBER, 1959

Annual subscription One Guinea, *post free; single copy 6 s*

THOMAS NELSON & SONS LTD.,
Parkside Works · Edinburgh 9